Lecture 1 (Tuesday, 8/11/09):
Interplay between superconductivity and magnetism in f-electron systems
(conventional superconductivity)

Lecture 2 (Wednesday, 8/12/09):
Unconventional superconductivity, magnetic and charge order, and quantum criticality in heavy fermion f-electron materials
(unconventional superconductivity)
f-electron materials

• f-electron materials – multinary compounds and alloys containing rare earth (R) and actinide (A) ions with partially-filled f-electron shells and localized magnetic moments

• Localized magnetic moments of R and A ions interact with the momenta and spins of the conduction electrons

• Certain R ions (Ce, Pr, Sm, Eu, Tm, Yb) and A ions exhibit valence instabilities; i.e., localized f-states hybridize with conduction electron states

• Competing interactions — readily “tuned” by x, P, H (“knobs”)

• Wide variety of correlated electron phenomena
  – Rich and complex phase diagrams in the hyperspace of T, x, P, H

• Situations involving competing interactions
  – One phenomenon survives at the expense of another
  – Interactions conspire to produce a new phenomenon (e.g., sinusoidally-modulated magnetic state that coexists with SC in FM superconductors)

• Brief survey – point of view of experimentalist

• Materials driven physics!
  • Materials – reservoir of new electronic states and phenomena
  • Opened up new research directions in condensed matter physics
f-electron materials
Examples

- Destruction of SC at $T_{c2} < T_{c1}$ (reentrant SC) due to Kondo effect
- Kondo effect (impurities and lattice)
- Destruction of SC at $T_{c2} < T_{c1}$ (reentrant SC) due to FM order
- Occurrence of sinusoidally modulated magnetic state ($\lambda \sim 100$ Å) that coexists with SC in FM superconductors
- Coexistence of SC & AFM order
- Coexistence of SC & itinerant FM
- Magnetic field induced SC (MFIS)
- Heavy fermion compounds ($m^* \sim 10^2 m_e$)
- Unconventional SC in heavy fermion compounds
  - Electron pairing with $L > 0$, nodes in SCing energy gap, magnetic pairing mechanism
- Occurrence of SC near magnetic QCPs accessed by pressure
- NFL behavior associated with QCPs
- Heavy fermion behavior and SC in PrOs$_4$Sb$_{12}$, possibly due to electric quadruple, rather than magnetic dipole, fluctuations

Lecture 1; Lecture 2
Lecture 1:

Interplay between superconductivity and magnetism in f-electron systems

Outline:

(1) f-electron materials
(2) Conventional superconductivity
(3) Magnetic interactions in superconductors
(4) Paramagnetic impurities in superconductors
(5) Magnetic field induced superconductivity (MFIS)
(6) Magnetic ordering via the RKKY interaction
(7) Magnetically ordered superconductors
Lecture 2:

Unconventional superconductivity, magnetic and charge order, and quantum criticality in heavy fermion f-electron materials

Outline:

(1) Unconventional superconductivity
(2) Heavy fermion compounds
(3) Superconductivity in heavy fermion compounds
(4) Competition between Kondo effect and RKKY interaction
(5) Non-Fermi liquid (NFL) behavior and quantum criticality
(6) SC near antiferromagnetic (AFM) quantum critical points (QCPs) under pressure
(7) Coexistence of SC and itinerant ferromagnetism
(8) Heavy fermion behavior and unconventional SC in PrOs$_4$Sb$_{12}$; evidence for electron pairing via electric quadrupole interactions
Local moment paramagnetism and magnetic order
**Local moment paramagnetism: magnetization**

\[ M = -g_J \mu_B J \]

\[ J = L + S \quad \text{(determined from Hund’s rules)} \]

\[ g_J = 1 + \frac{[J(J+1) + S(S+1) - L(L+1)]}{2J(J+1)} \quad \text{Landé g-factor} \]

\[ \mu_B = \frac{e\hbar}{2m c} = 0.927 \times 10^{-20} \text{ erg/gauss} \]

\[ E = m_J g_J \mu_B H; \ m_J = J, J-1, \ldots, -J \quad \text{(2J+1 equally spaced levels)} \]

\[ M = N g_J \mu_B B_J(x); \ x = g_J \mu_B H/k_B T \]

\[ B_J(x) = \left[ \frac{2J+1}{2J} \right] \text{ctnh}\left[ \frac{(2J+1)x}{2J} \right] - \left( \frac{1}{2J} \right) \text{ctnh}\left( \frac{x}{2J} \right) \quad \text{Brillouin function} \]
Local moment paramagnetism: magnetization

\[ M \approx [Ng^2J(J+1)\mu_B^2/3k_B T]H = (N\mu_{\text{eff}}^2/3k_B T)H = \chi(T)H \]

\[ \chi(T) = M/H = N\mu_{\text{eff}}^2/3k_B T = C/T \] \text{Curie law}

\[ C = N\mu_{\text{eff}}^2/3k_B \] \text{Curie constant}

\[ \mu_{\text{eff}} = \sqrt{g_J[J(J+1)]} \mu_B \] \text{Effective moment}

\[ x \ll 1 \]

\[ M \approx g_JJ\mu_B \] \text{Saturation moment}

\[ x \gg 1 \]

\[ \chi^{-1} \]

slop: \( C^{-1} = 3k_B/N\mu_{\text{eff}}^2 \)
S, L, J for lanthanide ion determined from Hund’s rules

Hund’s rules:

Lanthanide ion with configuration $4f^n$

4f electron: $l = 3$, $s = 1/2$

$S = \text{maximum value } \sum (s_z)_i$

$L = \text{maximum value } \sum (l_z)_i$ (subject to Pauli principle)

$J = |L-S|$ 4f shell less than half filled ($n < 7$)

$J = L+S$ 4f shell more than half filled ($n \geq 7$)

e.g., Ce$^{3+}$ ($4f^1$) $S = 1/2$, $L = 3$, $J = |L-S| = 5/2$

Pr$^{3+}$ ($4f^2$) $S = 1$, $L = 5$, $J = |L-S| = 4$

Gd$^{3+}$ ($4f^7$) $S = 7/2$, $L = 0$, $J = L+S = 7/2$ (so-called “S-state” ion)

Yb$^{3+}$ ($4f^{13}$) $S = 1/2$, $L = 3$, $J = L+S = 7/2$ (one f-“hole”)

$2J+1$ degeneracy of Hund’s rule ground state multiplet can be lifted by crystalline electric field (CEF) $\Rightarrow$ CEF ground and excited states

*Local moment paramagnetism: Hund’s rules*
Local moment magnetic ordering

\[ \chi(T) = \frac{N\mu_{\text{eff}}^2}{3k_B(T-\theta)} \]

Curie-Weiss law

\( \theta \) – Curie-Weiss temperature

Ferromagnetic order: \( \theta \approx \theta_f \) (Curie temperature)

Antiferromagnetic order: \( \theta \approx -T_N \) (Néel temperature)
Conventional superconductivity
Conventional superconductivity

Attractive interaction (electron-phonon)

\[ (k^\uparrow, -k^\downarrow) \quad \text{"Cooper pairs"} \]

\[ T_c \sim \Theta \exp[-1/N(0)V] \]

\[ \frac{F_n(0) - F_\pi(0)}{2N(0)\Delta(0)} = dx \]

\[ \Delta(0) = 1.764 k_B T_c \]

\[ \xi_0 = 0.18 \frac{\hbar v_F}{k_B T_c} \quad \text{coherence length} \]

Energy gap (order parameter)

\[ \frac{\Delta(T)}{\Delta(0)} \approx 1.74 \left(1 - \frac{T}{T_c}\right)^{1/2} \]

for \( T \approx T_c \)
Conventional superconductivity

Specific heat

\[ \frac{C}{kT_c} \]

\[ C = C_{es} + C_{ew} = \gamma T \]

\[ \frac{\Delta C}{kT_c} = 1.43 \]

\[ C_{es}/kT_c \approx a \exp(-bT_c/T) \]

\[ a = 8.5, \quad b = 1.44 (2.5 < T_c/T < 6) \]

\[ a = 26, \quad b = 1.62 (7 < T_c/T < 11) \]

Spin susceptibility

\[ \chi_{so} = 0 \]

\[ \chi_n = 2N(0)\mu_B^2 \]

\[ \chi_{so} \text{ finite} \]

\[ \chi_{so} = \infty \text{ (BCS)} \]

\[ \chi_{so} - \text{spin-orbit scattering} \]

Yosida (1958)
Conventional superconductivity

Type II superconductivity (compounds, alloys)

\[ \kappa = \frac{\lambda}{\xi} > 1/\sqrt{2} \]

- **Meissner effect**
  - \( H < H_{cl} \)

- **Vortex lattice**
  - \( H_{cl} < H < H_{c2} \)

**Fluxoid**
- **Core:**
  - Flux quantum \( \Phi_0 \)
  - Diameter \( \sim \xi \)
- **Supercurrent vortex:**
  - Diameter \( \sim \lambda \)
Superconducting-magnetic interactions

Superconductor containing R ions (magnetic moment $\mu$)

Electromagnetic

$\mathbf{m}(r) \rightarrow \mathbf{j}(r)$

Exchange

$\mu = g_J \mu_B J$

$\mathbf{H}_{\text{ex}} = -2J(g_J - 1)\mathbf{J} \cdot \mathbf{s}$

“Pair breaking” interactions

Conventional SC: $(\mathbf{k}^\uparrow, -\mathbf{k}^\downarrow)$
Paramagnetic impurities in superconductors
Interplay between superconductivity and magnetism: History

• Origin
  
  Theory – Ginzburg (57)
  Experiments – Matthias, Suhl, Corenzwit (58)

• Background (57-76)
  Experiments
  – Binary and pseudobinary R impurity systems; e.g., La$_{1-x}$R$_x$, Y$_{1-x}$R$_x$Os$_2$
  – Rapid depression of $T_c$ with $x$ ($x_{cr} \sim 1$ at% for La$_{1-x}$Gd$_x$)
  – Results – provocative, inconclusive wrt coexistence of two phenomena (chemical clustering, short-range or “glassy” magnetic order)

  Theory
  – Striking predictions, inapplicable to systems then under investigation

  Spin-off
  – Understanding of effects of paramagnetic impurities on superconductivity – CEF, Kondo effect, LSF, etc.

• Revival (~76)
  Experiments – Binary, ternary and quaternary R and U compounds
  ⇒ new, unusual physical effects and phenomena

  Theory – Intense activity
Paramagnetic impurities in superconductors

Matthias, Suhl, Corenzwit (58) —
• $T_c(x)$ for $La_{1-x}R_x$
  $R = Gd$: $T_c(x) \rightarrow 0$ K for $x \approx 1$ at.%

Matthias, Suhl, Corenzvit (58) —
• Depression of $T_c$ for $x=1$ at.%,
  $-\Delta T_c = T_{co} - T_c$, correlates with $S$ of $R$ solute

Herring (58); Suhl & Matthias (59) —
Exchange interaction: $H_{int} = -2(g_J - 1)JJ \cdot s$
$\Delta T_c \propto J^2(g_J - 1)^2J(J+1)$; deGennes factor $\equiv (g_J - 1)^2J(J+1)$

NOTE: Ce
Paramagnetic impurities in superconductors

Anomalous depression of $T_c$ for Ce $\Rightarrow$ hybridization of Ce localized 4f and itinerant electron states $\Rightarrow$ $J \sim -\langle V_{kf}^2 \rangle / E_f < 0$ $\Rightarrow$ Kondo effect

LaCe: Sugawara, Eguchi (66); (LaCe)Al$_2$: Maple, Fisk (68)

LaRE (T$_{co}$ = 6 K)
Matthias, Suhl, Corenzwit (58)
(LaRE)Al$_2$ (T$_{co}$ = 3.3 K)
Maple (70)
$J \sim 0.1$ eV
Paramagnetic impurities in superconductors

Pair breaking ⇒ rapid suppression of SC
Two cases: 

- $J > 0$ (ferromagnetic)
- $J < 0$ (antiferromagnetic)

\[
\frac{T_c}{T_0} = \ln \left( \frac{\alpha}{\alpha_{cr}} \right) \quad \text{Abrikosov & Gor'kov (1961-1962)}
\]

$\alpha$ - pair breaking parameter

\[
\alpha = \frac{1}{\lambda_\text{ex}} = \hbar^2 \tau N(0) \int \frac{d^2 q}{(2\pi)^2} (q^2 - 1)^2 J (J+1)
\]

\[
\alpha_{cr} = \frac{k_B T_c}{4\pi^2} \quad (\ln \frac{\lambda}{\alpha_{cr}} = 0.57721 - \text{Euler's const.})
\]

Explicitly -

\[
\ln \left( \frac{T_c}{T_0} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + 0.17 \frac{\alpha}{\alpha_{cr}} \frac{T_c}{T_0} \right)
\]

$\psi$ - digamma function

Limit $\alpha \to 0$ -

\[
\frac{T_c}{T_0} = 1 - 0.691 (\frac{\alpha}{\alpha_{cr}}) = 1 - 0.691 (\frac{\lambda}{\alpha_{cr}})
\]
Paramagnetic impurities in superconductors

Linear region \( n/n_{cr} < 1 \)

\[
\frac{dT_c}{dn} \bigg|_{n=0} = -(\pi^2/2)h^2 N(0) \tilde{g}^2 (g_f - 1)^2 J(J+1)
\]

de Gennes's factor (maximum at 6d)

Other predictions -

\[
\frac{\Delta c}{\Delta c_0} = \sqrt{n} \left( \frac{T_c}{T_c} \right).
\]

deviates from BCS law of corresponding states \( \frac{\Delta c}{\Delta c_0} = \frac{T_c}{T_c} \)

Gapless \( SC \) \( \Delta \rightarrow 0 \)

\( \Omega \rightarrow 0 \) faster than \( \Delta \rightarrow 0 \) with \( \Omega \)
Gapless superconductivity
$La_{1-x}Gd_xAl_2$; $T_c$ vs $x$ phase boundary

$T_{co} = 3.3$ K
$n_{cr} = 0.59$ at.% Gd

"Gapless" superconductivity

*Woolf, Reif (65) - tunneling Pb$_{1-x}$Gd$_x$*

*Finnemore et al. (65) - specific heat La$_{1-x}$Gd$_x$*

*M. B. Maple (68)*
La$_{1-x}$Gd$_x$Al$_2$; specific heat jump vs $T_c$

W. R. Decker, D. K. Finnemore (68)
C. A. Luengo, M. B. Maple (73)
Kondo effect in superconductors

SCing metal containing paramagnetic impurities (spin S)

- **AFM exchange interaction**
  \[ H_{\text{ex}} = -2J \mathbf{S} \cdot \mathbf{s} \quad \text{with } J < 0 \]

- **Formation of many body singlet state below** \( T_K \)
  \[ T_K \sim T_F \exp(-1/N(E_F)|J|) \quad \text{or } T_K \rightarrow \text{effective } T_F \]

- **Normal State**
  - \( T > T_K \): Local moment behavior
    \[ \chi(T) \sim N \mu_{\text{eff}}^2/3k_B(T-\theta) \quad \text{where } \theta \sim -3T_K \]
    \[ \rho(T) \sim -\ln T \quad \text{"resistivity minimum"} \]
  - \( T \ll T_K \): Many body singlet
    Nonmagnetic heavy Fermi liquid (FL)
    \[ \chi(T) \propto \gamma(T) = C(T)/T \sim \text{const.} \]
    \[ \rho(T) \approx \rho(0)[1-(T/T_K)^2] \]
Kondo effect in superconductors

- Superconducting state

  Competition between:
  1. Singlet spin paired \((k↑,-k↓)\) SCing state \((E_{SC} \sim k_B T_c)\);
  2. Kondo many body singlet state \((E_K \sim k_B T_K)\)

    - \(T_K \ll T_{co}\): Reentrant \(T_c(x)\) curve!
    - \(T_K \gg T_{co}\): Exponential-like depression of \(T_c\) with \(x\)
    - \(T_K \approx T_{co}\): Maximum in initial rate of depression of \(T_c\)

Theory: Müller-Hartmann, Zittartz (70-71); Zuckermann (68): Ludwig, Zuckermann (71)
\[ \frac{T_c}{T_c'} = \ln \left( \frac{\alpha}{\alpha_{cr}} \right) \]

\[ \alpha / \alpha_{cr} = n B \left\{ \frac{\pi^2 S(S+1)}{\ln^2(T/T_c) + \pi^2 S(S+1)} \right\} \]

Müller-Hartmann & Zittartz (MHZ - 1971)

Other predictions -

\[ \Delta C / \Delta C_0 \text{ vs } T_c / T_c' \text{ deviates from both BCS law of corresponding states & AG theory} \]

Bound state in energy gap
Kondo effect in $\text{La}_{1-x}\text{Ce}_x\text{Al}_2$: electrical resistivity

$T_K \sim 0.1$ K

Magnetic scattering
“Kondo” contribution to electrical resistivity:
$\Delta \rho(T) = \rho(x,T) - \rho(0,T)$

Maple, Fisk (68)
Kondo effect in La$_{1-x}$Ce$_x$Al$_2$: specific heat

Curve d - theory of Bloomfield and Hamann for $S = 1/2$ and $T_K = 0.42$ K (H = 0)

S. D. Bader, N. E. Phillips, M. B. Maple, C. A. Luengo (75)
Ce has ground state doublet and excited state quartet with splitting $\delta \sim 100$ K in CEF.

$\theta_p \approx -0.5$ K $\Rightarrow T_K \approx 0.1$ K
Kondo effect in La$_{1-x}$Ce$_x$Al$_2$: reentrant $T_c$ vs $x$ curve

- Kondo effect: $T_K << T_{co}$ “reentrant SC”
- Riblet, Winzer (71) (U. Köln)
- Maple, Fertig, Mota, DeLong, Wohlleben, Fitzgerald (72) (UCSD)

Maple (68)
Kondo effect in La$_{1-x}$Ce$_x$Al$_2$: specific heat jump vs $T_c$

- C. A. Luengo, M. B. Maple, W. A. Fertig (72)
- H. Armbrüster, F. Steglich (73)
- S. D. Bader, N. E. Phillips, M. B. Maple, C. A. Luengo (73)
Kondo effect in $\text{Th}_{1-x}\text{U}_x$: electrical resistivity

- $\text{Th}_{1-x}\text{U}_x$: conventional Kondo effect (Fermi liquid - low $T$)
  - $\Delta \gamma \approx 270 \text{ mJ/mol U-K}^2$
  - $\Delta \rho (T) = \rho_0 [1 - (T/T_K)^2]; \ T_K \approx 100 \text{ K}$

M. B. Maple et al. (70)
Kondo effect in $\text{Th}_{1-x}\text{U}_x$: magnetic susceptibility and thermopower

$\text{Th}_{1-x}\text{U}_x$:  
- $\chi(T) = C/(T-\theta)$  
- $\theta \approx -3 \ T_K$  
- $C = N\mu_{\text{eff}}^2/3k_B$

- Peak in thermoelectric power  
  $\Rightarrow T_K \approx 100 \ K$

M. B. Maple et al. (70)
Kondo effect in $\text{Th}_{1-x}\text{U}_x$: exponential $T_c$ vs $x$ curve

Comparison of $T_c$ vs $x$ curves of $\text{Th}_{1-x}\text{U}_x$, $\text{Al}_{1-x}\text{Mn}_x$, $\text{Th}_{1-x}\text{Ce}_x$

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_o$ (K)</th>
<th>$T_c$ (K)</th>
<th>$n_o (a/o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ThU</td>
<td>$\sim$100</td>
<td>1.362</td>
<td>0.159</td>
</tr>
<tr>
<td>AlMn</td>
<td>$\sim$500</td>
<td>1.169</td>
<td>0.174</td>
</tr>
<tr>
<td>ThCe</td>
<td>$\sim$1000</td>
<td>1.362</td>
<td>3.07</td>
</tr>
</tbody>
</table>

$T_c / T_{c_0} = \exp \left[ -An / (1-Dn) \right]$

Kondo effect: $T_K >> T_{c_0}$

Maple, Huber, Coles, Lawson (70); Huber, Maple (70)
Comparison of $\Delta C/\Delta C_0$ vs $T_c/T_{co}$ curves of $\text{Th}_{1-x}U_x$, $\text{Al}_{1-x}\text{Mn}_x$, $\text{Th}_{1-x}\text{Ce}_x$

$T_c << T_K$: Conforms to BCS law of corresponding states $\Delta C/\Delta C_0 = T_c/T_{co}$

- (○) C. A. Luengo, J. M. Cotignola, J. Sereni, A. R. Sweedler, M. B. Maple, J. G. Huber (72)
- (●) H. L. Watson, D. T. Peterson, D. K. Finnemore (73)
- (□) D. L. Martin (61)
- (■) F. W. Smith (72)
- (▲) C. W. Dempesey (70)
Pressure-induced demagnetization of Ce impurities in La

- $|J|$ & $T_K$ increase with $P$  
  $\Rightarrow T_K/T_{co}$ increases with $P$ from $<< 1$ to $>> 1$ through $T_K/T_{co} \approx 1$ at $\sim 15$ kbar  
  $\Rightarrow$ maximum in $\Delta T_c$ at $\sim 15$ kbar

- Analogue of Ce $\gamma-\infty$ transition

- Maple, Wittig, Kim (69)
Demagnetization of Ce impurities in $(La_{1-y}Th_y)_{1-x}Ce_x$

Th substitution acts like pressure!

Huber, Fertig, Maple (72)
Paramagnetic impurities in superconductors: summary

I. Long-Lived Local Moments
   A. Weak Itinerant-Local Electron Mixing - J > 0; Temperature Independent Pair Breaking
      Exemplary Systems - (La,Gd)Al₂
         \[ \text{BCS} \]
         \[ \text{AG} \]
      \[ \frac{T_c}{T_{c_0}} \quad \text{n} \]

   B. Moderate Itinerant-Local Electron Mixing - J < 0 and \( T_{c_0} >> T_K \); Temperature Dependent Pair Breaking
      Exemplary System - (La,Ce)Al₂
         \[ \text{BCS} \]
         \[ \text{AG} \]
Paramagnetic impurities in superconductors: summary

II. Short-Lived Local Moments

Strong Itinerant-Local Electron Mixing - $T_c \ll T_0$; Pair Weakening

Exemplary Systems:
- AlMn
- ThU
- ThCe

Graphs showing $T_c/T_{c0}$ and $\Delta C/\Delta C_0$ versus $n$ and $T_c/T_{c0}$.
Magnetic field induced superconductivity (MFIS)
Origin of the upper critical field $H_{c2}$

1. **Orbital Critical Field $H_{c2}^*$**

   $$H_{c2}^* = \frac{\Phi_0}{2\pi R^2} \quad \Phi_0 = h c / 2e$$

2. **Paramagnetic Limiting Field $H_p$**

   - Normal state
     $$F_n(H) = F_n(0) - \frac{1}{2} \chi_n H^2$$
   - Superconducting state
     $$F_s(H) = F_s(0) - \frac{1}{2} \chi_s H^2$$

   First order transition from superconducting to normal state when
   $$F_n(H_p) - F_s(H_p) = 0$$
   $$= [F_n(0) - F_s(0)] - \frac{1}{2} \left( \chi_n - \chi_s \right) H_p^2$$
   $$= \frac{1}{2} N(0) \Delta^2 - \frac{1}{2} \left( \chi_n - \chi_s \right) H_p^2$$

\[
\begin{align*}
\frac{e}{mc} (\mathbf{A} \cdot \mathbf{A}) &= \frac{eA}{mc} (\mathbf{A} \cdot \mathbf{A}) \rightarrow H_{c2}^* \\
\mu_b \cdot H &= -g \mu_b (\mathbf{z} \cdot \mathbf{H}) \rightarrow H_p
\end{align*}
\]
Origin of the upper critical field $H_{c2}$

To increase $H_{c2}$ above $H_p$
- Increase spin-orbit scattering $\Rightarrow$ increase $\chi_s$
- Compensate applied field with exchange field $\Rightarrow$ MFIS

$$H_p = \left[ \frac{N(0)}{\chi_n - \chi_s} \right]^{1/2} \cdot \Delta$$

$\chi_s = 0$
$\chi_n = \frac{1}{2} \frac{N(0) g^2}{\mu_B}$

$$H_{p0} = \frac{\sqrt{2} \Delta(0)}{g \mu_B} = 18.4 \frac{T_c}{k_B} (kOe)$$

Clogston-Chandrasekhar Limit (1962)
Magnetic field induced superconductivity

SCHEME FOR RAISING PARAMAGNETICALLY LIMITED $H_{c2}$

$(Eu_{1-x}M_x)Mo_6S_8$ where $M = Sn, Pb, La, Ho, Yb$

$H_{c2}$ determined by two interactions which "break" superconducting electron pairs ($k\uparrow;k\downarrow$):

1. $p \cdot A = \hbar(k \cdot A) \rightarrow$ orbital critical field $H_{c2}^*$

2. $-\mu \cdot H = -2\mu_B(\hat{s} \cdot \hat{H}) \rightarrow$ paramagnetic limiting field $H_p$

Assume $H_{c2} \sim H_p < H_{c2}^*$

$Eu^{2+} \Rightarrow J = S = 7/2 \Rightarrow \mu = 7\mu_B$

$H_{ex} = -2\hat{J} \cdot \hat{S} = -\mu \cdot H_{ex}^l$ where $\mu = 2\mu_B \hat{s}$, $H_{ex}^l = (\hat{J}/\mu_B)\hat{s}$

$\hat{H}_T = \hat{H} + H_{ex}^l$ where $H_{ex} = nH_{ex}^l$

$\hat{J} < 0 \Rightarrow H_T = H - |H_{ex}| < H$

$H_{ex} \approx <S_z> = SB_{7/2}(7\mu_B H/k_B T)$ (Brillouin function)
Magnetic field induced superconductivity

SC for $|H_T| = |H - |H_{ex}|| \leq H_p$ or $|H_{ex}| - H_p \leq H \leq H_{ex} + H_p$

$\Rightarrow$

(1) Enhancement of $H_{C2}$;

(2) Magnetic field induced SC!

FM with $\oint < 0$ in $H \rightarrow$ SC! (Jaccarino & Peter, '62)

Not yet been observed!
Magnetic field induced superconductivity

\[ H_T = H - |H_J| \]
Magnetic field induced superconductivity

\[ \phi \]

Fisher et al. (83)

\[ \text{Sn}_{0.25}\text{Eu}_{0.75}\text{Mo}_6\text{S}_{7.2}\text{Se}_{0.8} \]

\[ \text{S} \]

\[ \text{N} \]

\[ \text{APPLIED MAGNETIC FIELD H (TESLA)} \]

\[ \text{UPPER CRITICAL MAGNETIC FIELD } H_{c2} \text{ (TESLA)} \]

\[ \text{TEMPERATURE T (K)} \]
Magnetic ordering via RKKY interaction

\[ H_{\text{int}} = -2J (g_f-1) \mathbf{J} \cdot \mathbf{S}(r) \]
\[ = -\left[ 2 (g_f-1) \frac{g}{\mu_B} \right] \mathbf{J} \cdot \mathbf{S}(r) \cdot (\mu_B \mathbf{S}) \]
\[ = -\frac{H_{\text{eff}}(r)}{m} \mathbf{m} \]

\[ H_{\text{eff}}(r) = \left[ 2 (g_f-1) \frac{g}{\mu_B} \right] \mathbf{J} \cdot \mathbf{S}(r) \]

\[ H_{\text{eff}}(q) = \left[ 2 (g_f-1) \frac{g}{\mu_B} \right] \mathbf{J} \cdot \mathbf{S}(q) \]

\[ \Sigma(q) = \frac{\mathbf{M}(q)/g \mu_B}{2} \frac{1}{2} \sum_{k<k_F} \mathbf{J} \cdot \mathbf{S}(q) \cdot e^{i \mathbf{q} \cdot \mathbf{r}} \]

\[ = \left[ 2 (g_f-1) \frac{g}{g_B^2} \right] \mathbf{J} \cdot \mathbf{S}(q) \cdot e^{i \mathbf{q} \cdot \mathbf{r}} \]

\[ \chi(q) = \chi_F \left( \frac{q}{2k_F} \right) \]

\[ = \chi_F \left[ \frac{1}{2} + \frac{k_F}{2q} (1 - \left( \frac{q^2}{2k_F} \right)^2) \log \left| \frac{2k_F q}{2k_F - q} \right| \right] \]

where \[ \chi_F = 2N(0) \mu_B^2 \]

\[ \Sigma(q) \sim \left\{ \frac{\sin(2k_F r) - 2k_F r \cos(2k_F r)}{(2k_F r)^2} \right\} \sim \frac{\cos(2k_F r)}{(2k_F r)^3} \text{ for } k_F \ll 1 \]
Magnetic ordering via RKKY interaction

leads to ferromagnetic, antiferromagnetic, or complicated magnetic structures -

\[ H_{RKKY} = -\frac{4(\eta - 1)^2 g^2}{g^2 \mu_B^2 V} \sum_{\ell=\pm 1} \epsilon_{\ell} e^{\frac{i \ell \cdot \xi}{\xi}} J_{i} \cdot J_{j} \]

ELECTRON BAND STRUCTURE
SUPERCONDUCTIVITY
Magnetically ordered superconductors
Magnetically ordered superconductors

- **Superconducting ternary R Compounds (ordered R sublattice)**
  - $\text{RMo}_6\text{S}_8$ \(\text{Fischer, Treyvaud, Chevrel, Sergent (75)}\)
  - $\text{RMo}_6\text{Se}_8$ \(\text{Shelton, McCallum, Adrian (76)}\)
  - $\text{RRh}_4\text{B}_4$ \(\text{Matthias, Corenzwit, Vandenberg, Barz (77)}\)

- **Antiferromagnetic superconductors**
  
  **Coexistence of SC & AFM**
  - $\text{RMo}_6\text{Se}_8$, R = Gd, Tb, Er \(\text{UCSD (77)}\)
  - $\text{RMo}_6\text{S}_8$, R = Gd, Tb, Dy, Er \(\text{U. Geneva (77)}\)
  - $\text{RRh}_4\text{B}_4$, R = Nd, Sm, Tm \(\text{UCSD (79)}\)
  - $\text{RNi}_2\text{B}_2\text{C}$ \(\text{Nagarajan et al. (94); Cava et al. (94)}\)
  - $\text{RNi}_2\text{B}_2\text{C}$ (single crystals) \(\text{Canfield et al. (94)}\)

- **Ferromagnetic superconductors**
  
  **Destruction of SC by onset of FM at** \(T_{c2} \sim \theta_C < T_{c1}\)
  
  **SC-FM interactions** – sinusoidally-modulated magnetic state (\(\lambda \sim 100\,\text{Å}\)) that coexists with SC near \(T_{c2}\)
  - $\text{ErRh}_4\text{B}_4$ \(\text{UCSD (77)}\)
  - $\text{HoMo}_6\text{S}_8$ \(\text{U. Geneva (77)}\)
  - $\text{ErRh}_{1.1}\text{Sn}_{3.6}$ \(\text{AT&T, UCSD, BNL (80)}\)
Two weakly interacting subsystems:
RhB “molecular units” or clusters $\rightarrow$ SC
R magnetic moments $\rightarrow$ magnetic order
Comparable values of $T_c$ and $T_M$
Superconducting and magnetic ordering temperatures of RRh$_4$B$_4$ compounds

After M. B. Maple, H. C. Hamaker, L. D. Woolf (82)
B. T. Matthias, E. Corenzwit, J. M. Vandenberg, H. Barz (77)
* Ce, Pr – T. Ooyama, K. Kumagai, J. Nakajima, M. Shimotomai (87)
$H_{c2}(T)$ of $RRh_4B_4$ magnetic superconductors

- Nonmagnetic SC: LuRh$_4$B$_4$
- AFM-SCs: NdRh$_4$B$_4$, SmRh$_4$B$_4$
- FM-SC: ErRh$_4$B$_4$
Magnetic superconductor NdRh$_4$B$_4$: resistive transition curves

Magnetic superconductor \( \text{NdRh}_4\text{B}_4 \): neutron scattering

Reentrant SC due to FM order: $\text{ErRh}_4\text{B}_4$

- Fertig, Johnston, DeLong, McCallum, Maple, Matthias (77)
- Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80) (neutron scattering)

SC & sinusoidally-modulated magnetic state ($\lambda \sim 100 \text{ Å}$)
Reentrant FM SC $\text{ErRh}_4\text{B}_4$: specific heat

Macroscopic coexistence of SC & normal FM domains

Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80)
Microscopic coexistence of SC & sinusoidally-modulated magnetic state with λ~100 Å

Small angle neutron scattering

\[ T_{c2} \approx 0.65 \text{ K} \]

Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80)

Similar behavior – HoMo\(_6\)S\(_8\)

Lynn et al. (81)
LINEARLY POLARIZED SINUSOIDALLY MODULATED MAGNETIC STATE ($\mu \perp c$) — ErRh$_4$B$_4$

Neutron scattering experiments on ErRh$_4$B$_4$ single crystal

Sinha, Crabtree, Hinks, Mook (81)

Theories: e.g.,

• Anderson, Suhl (59)
• Suhl (78)
• Blount, Varma (79)
• Bulaevski, Rusinov, Kulik (79)
• Matsumoto, Umezawa, Tachiki (79)
\((\text{Er}_{1-x}\text{Ho}_x)\text{Rh}_4\text{B}_4): \text{FM} - \mu \perp c \text{ vs } \mu \parallel c\)

\[ \text{PARAMAGNETIC} \]

\[ \text{SUPERCONDUCTING} \]

\[ \text{SC SINUSOIDALLY MODULATED} \]

\[ \& \text{NORMAL FM REGIONS} \]

\[ \text{FERROMAGNETIC (} \mu \parallel c \text{)} \]

\[ \text{MIXED} \]

\[ \text{FM (} \mu \perp c \text{)} \]

Johnston, Fertig, Maple, Matthias (78); Mook, Koehler, Maple, Fisk, Johnston, Woolf (82)
$\text{Ho}(\text{Rh}_{1-x}\text{Ir}_x)\text{B}_4$; FM vs AFM

• H. C. Ku, F. Acker, B. T. Matthias (80)
• K. N. Yang, S. E. Lambert, H. C. Hamaker, M. B. Maple, H. A. Mook, H. C. Ku (82)
• S. E. Lambert, M. B. Maple, O. A. Pringle, H. A. Mook (85)
**Oscillatory magnetic state in FM SCs**

ErRh₄B₄, HoMo₆S₈: $\lambda \sim 10^2 \quad \text{Å (neutron scattering)}$

Explanation based on electromagnetic interaction

*Explanation based on electromagnetic interaction*

- Blount & Varma (1979)
- Ferrell, Battacharjee & Bagchi (1979)
- Matsuda, Umezawa & Tachiki (1979)
- Suhl (1980)

\[
\tilde{h}_m(x) = \gamma(-i\nabla)m(x) + \tilde{h}(x)
\]

$\tilde{h}_m(x)$: molecular field acting on RE iron

$m(x)$: RE magnetic moment

$\tilde{h}(x)$: magnetic field generated by persistent current

\[
\tilde{h}_m(x) = \gamma(-i\nabla)m(x) + \tilde{h}(x) = \gamma c(x)m(x) - 4\pi F c(x)m(x)
\]

\[
= [\gamma c(x) - 4\pi F c(x)] m(x) = \gamma c(x)m(x)
\]
Oscillatory magnetic state in FM SCs

Normal:

\[
\chi(q) = \frac{(T - Dq^2)}{C}
\]

- \( T \) - Curie temperature
- \( D \) - magnetic stiffness coefficient
- \( C = N\mu_{\text{eff}}^2 / 8k_B \) - Curie constant

\( \chi(q) \) maximum at \( q = 0 \) \( \Rightarrow \lambda = \infty \)

\( \Rightarrow \) FM for \( T \leq T_m \) (2nd order)

Superconducting:

\[
\tilde{\chi}(q) = \chi(q) - 4\pi F(q) \ ; \ F(q) = \frac{\exp(-\xi q^2 / 2)}{\lambda^2 q^2 + \exp(-\xi q^2 / 2)}
\]

- \( \xi \) - coherence length
- \( \lambda \) - London penetration depth

\( \tilde{\chi}(q) \) maximum at \( Q \sim (4\pi C / D) \lambda \)

SC screens exchange interaction at long wavelengths

\( \Rightarrow \) AFM + SC for \( T_c \leq T < T_m \) (2nd order)

FM for \( T \leq T_{c2} \) (1st order)
Oscillatory magnetic state in FM SCs
Superconductivity

- Conventional superconductivity

  Electron pairs (Cooper pairs) – \((k^\uparrow, -k^\downarrow)\)
  
  \[ S = 0 \text{ (singlet)} \]
  
  \[ L = 0 \text{ (s-wave)} \]

  Pairing mechanism — electron-phonon interaction

  \[ T_c \approx \theta_D \exp(-1/N(E_F)V) \]

  Isotropic energy gap \(\Delta(k) = \Delta\)

  “Activated” behavior;
  
  \[ e.g., C_e(T) \sim \exp(-\Delta/T) \]
Superconductivity

- **Unconventional superconductivity**

  Electron pairs \((L > 0)\)
  
  \(S = 1\) (triplet) \(\Rightarrow\) \(L = 1\) (p-wave)
  
  \(S = 0\) (singlet) \(\Rightarrow\) \(L = 2\) (d-wave)

  Pairing mechanism – AFM spin fluctuations

  Anisotropic energy gap \(\Delta(k) \neq \Delta\)

  \(\Delta(k)\) vanishes at points or lines on Fermi surface

  “Power Law” behavior;

  e.g., \(C_e(T) \sim T^n\) (\(n = 2\), line nodes; \(n = 3\), point nodes)