ICMR Summer School on Novel Superconductors

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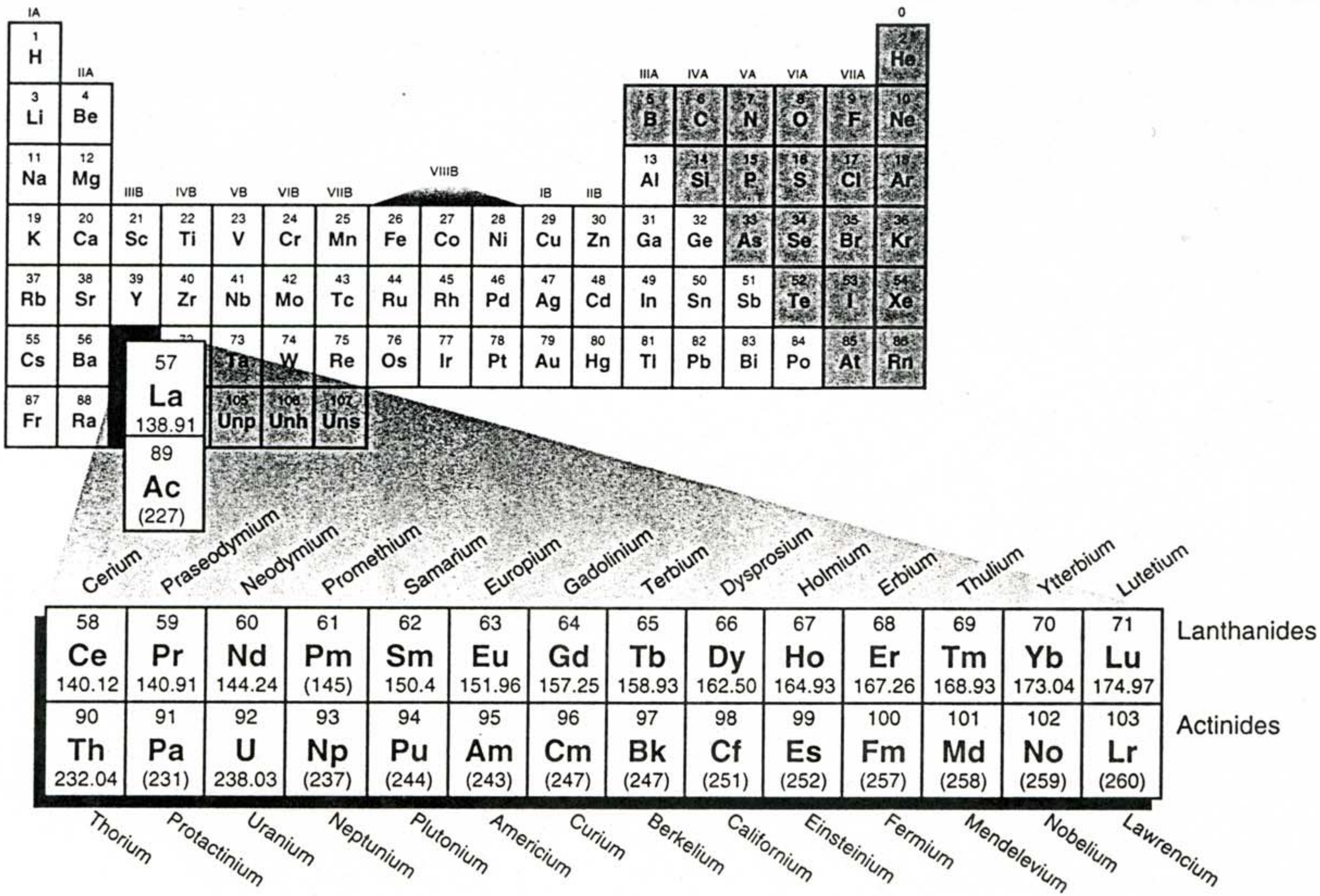
Lecture 1 (Tuesday, 8/11/09):
Interplay between superconductivity and magnetism in f-electron systems
(conventional superconductivity)

Lecture 2 (Wednesday, 8/12/09):
Unconventional superconductivity, magnetic and charge order, and quantum criticality in heavy fermion f-electron materials
(unconventional superconductivity)

f-electron materials

- f-electron materials – multinary compounds and alloys containing rare earth (R) and actinide (A) ions with partially-filled f-electron shells and localized magnetic moments
- Localized magnetic moments of R and A ions interact with the momenta and spins of the conduction electrons
- Certain R ions (Ce, Pr, Sm, Eu, Tm, Yb) and A ions exhibit valence instabilities; i.e., localized f-states hybridize with conduction electron states
- Competing interactions — readily “tuned” by x, P, H (“knobs”)
- Wide variety of correlated electron phenomena
  - Rich and complex phase diagrams in the hyperspace of T, x, P, H
- Situations involving competing interactions
  - One phenomenon survives at the expense of another
  - Interactions conspire to produce a new phenomenon (e.g., sinusoidally-modulated magnetic state that coexists with SC in FM superconductors)
- Brief survey – point of view of experimentalist
- Materials driven physics!
  - Materials – reservoir of new electronic states and phenomena
  - Opened up new research directions in condensed matter physics

f-electron materials



Examples

- Destruction of SC at $T_{c2} < T_{c1}$ (reentrant SC) due to Kondo effect &
- Kondo effect (impurities and lattice) &
- Destruction of SC at $T_{c2} < T_{c1}$ (reentrant SC) due to FM order &
- Occurrence of sinusoidally modulated magnetic state (λ ~ 100 Å) that coexists with SC in FM superconductors &
- Coexistence of SC & AFM order &
- Coexistence of SC & itinerant FM &
- Magnetic field induced SC (MFIS) &
- Heavy fermion compounds ($m^* \sim 10^2\, m_e$) &
- Unconventional SC in heavy fermion compounds &
  - Electron pairing with L > 0, nodes in SCing energy gap, magnetic pairing mechanism
- Occurrence of SC near magnetic QCPs accessed by pressure &
- NFL behavior associated with QCPs &
- Heavy fermion behavior and SC in $PrOs_4Sb_{12}$, possibly due to electric quadruple, rather than magnetic dipole, fluctuations &

& Lecture 1; & Lecture 2

Lecture outlines

Lecture 1:

Interplay between superconductivity and magnetism in f-electron systems

☐Outline:

(1) f-electron materials

(2) Conventional superconductivity

(3) Magnetic interactions in superconductors

(4) Paramagnetic impurities in superconductors

(5) Magnetic field induced superconductivity (MFIS)

(6) Magnetic ordering via the RKKY interaction

(7) Magnetically ordered superconductors

Lecture 2:

Unconventional superconductivity, magnetic and charge order, and quantum criticality in heavy fermion f-electron materials

Outline:

(1) Unconventional superconductivity

(2) Heavy fermion compounds

(3) Superconductivity in heavy fermion compounds

(4) Competition between Kondo effect and RKKY interaction

(5) Non-Fermi liquid (NFL) behavior and quantum criticality

(6) SC near antiferromagnetic (AFM) quantum critical points (QCPs) under pressure

(7) Coexistence of SC and itinerant ferromagnetism

(8) Heavy fermion behavior and unconventional SC in $PrOs_4Sb_{12}$; evidence for electron pairing via electric quadrupole interactions

Local moment paramagnetism and magnetic order

Local moment paramagnetism: magnetization

$\mathbf{M} = -g_J \mu_B \mathbf{J}$

$\mathbf{J} = \mathbf{L} + \mathbf{S}$ (determined from Hund's rules)

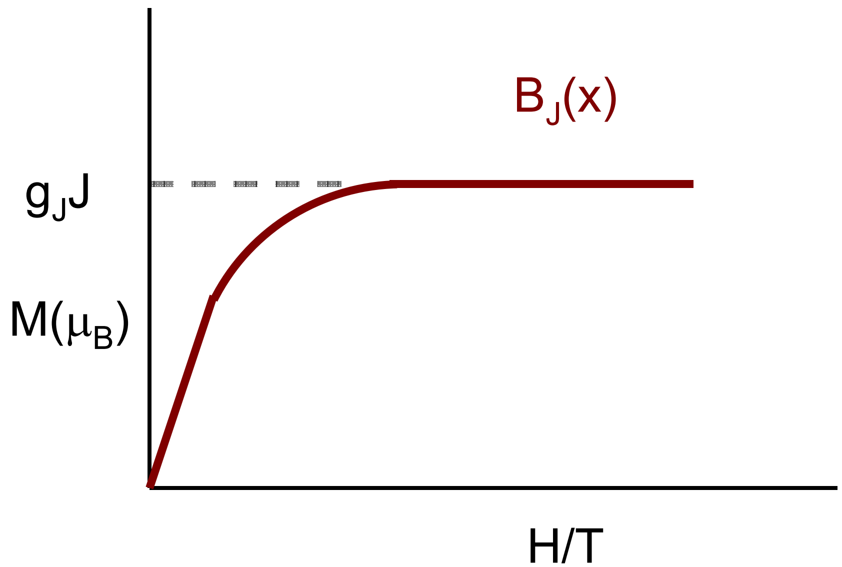
$g_J = 1 + [J(J+1) + S(S+1) - L(L+1)]/2J(J+1)$ Landé g-factor

$\mu_B = e\hbar/2mc = 0.927 \times 10^{-20}$ erg/gauss

$E = m_J g_J \mu_B H;\ m_J = J, J-1, \ldots, -J$ (2J+1 equally spaced levels)

$M = N g_J J \mu_B B_J(x);\ \ x = g_J J \mu_B H/k_B T$

$B_J(x) = [(2J+1)/2J]\,\mathrm{ctnh}[(2J+1)x/2J] - (1/2J)\,\mathrm{ctnh}(x/2J)$ Brillouin function



Local moment paramagnetism: magnetizaation

<u>x << 1</u>

$$M \approx [N g_J^2 J(J+1)\mu_B^2/3k_B T]H = (N\mu_{eff}^2/3k_B T)H = \chi(T)H$$

$$\boxed{\chi(T) = M/H = N\mu_{eff}^2/3k_B T = C/T}$$ Curie law

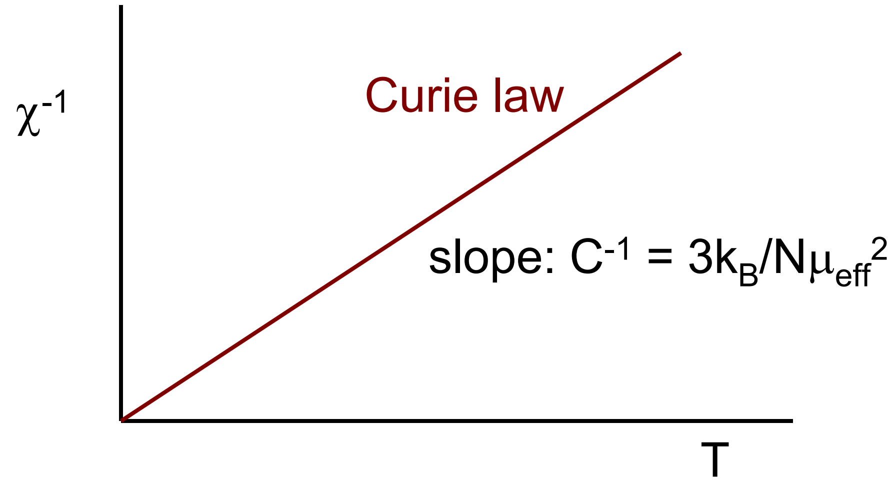
$C = N\mu_{eff}^2/3k_B$ Curie constant

$\mu_{eff} = g_J[J(J+1)]^{1/2}\mu_B$ Effective moment

<u>x >> 1</u>

$M \approx g_J J \mu_B$ Saturation moment

<u>x << 1</u>



Local moment paramagnetism: Hund's rules

S, L, J for lanthanide ion determined from Hund's rules

Hund's rules:

Lanthanide ion with configuration $4f^n$

4f electron: l = 3, s = 1/2

S = maximum value $\Sigma(s_z)_i$

L = maximum value $\Sigma(l_z)_i$ (subject to Pauli principle)

J = |L-S| 4f shell <u>less</u> than half filled ($n < 7$)

J = L+S 4f shell <u>more</u> than half filled ($n \geq 7$)

e.g., Ce^{3+} ($4f^1$) S = 1/2, L = 3, J = |L-S| = 5/2

Pr^{3+} ($4f^2$) S = 1, L = 5, J = |L-S| = 4

Gd^{3+} ($4f^7$) S = 7/2, L = 0, J = L+S = 7/2 (so-called "S-state" ion)

Yb^{3+} ($4f^{13}$) S = 1/2, L = 3, J = L+S = 7/2 (one f-"hole")

2J+1 degeneracy of Hund's rule ground state multiplet can be lifted by crystalline electric field (CEF) $\Rightarrow$ CEF ground and excited states

Local moment magnetic ordering

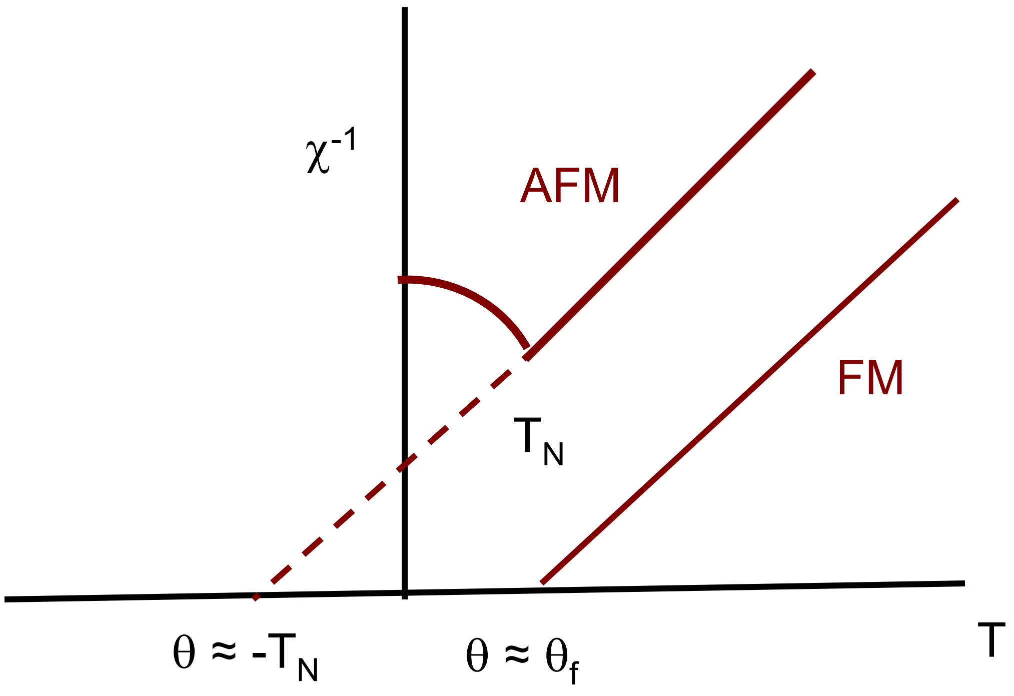
$$\chi(T) = N\mu_{eff}^2/3k_B(T-\theta)$$

Curie-Weiss law

θ – Curie-Weiss temperature

Ferromagnetic order: $\theta \approx \theta_f$ (Curie temperature)

Antiferromagnetic order: $\theta \approx -T_N$ (Néel temperature)



Conventional superconductivity

Conventional superconductivity

Attractive interaction (electron-phonon)

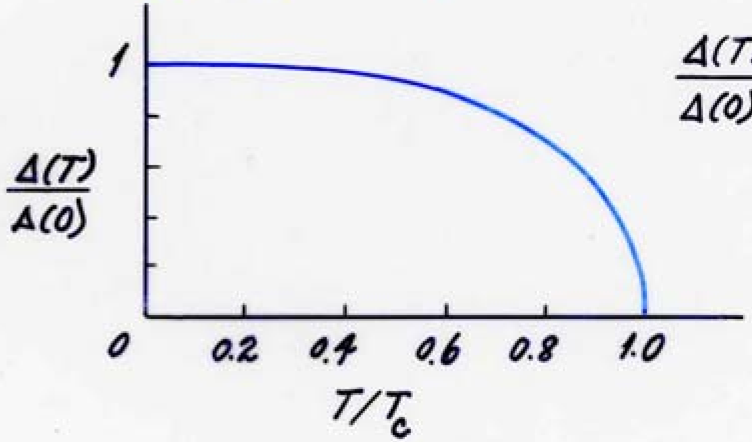
$(\underset{\sim}{k}\uparrow, -\underset{\sim}{k}\downarrow)$ "Cooper pairs"

$T_c \sim \theta_D \exp[-1/N(0)V]$

$F_n(0) - F_s(0) = \frac{1}{2} N(0) \Delta^2(0)$ $\Delta(0) = 1.764\, k_B T_c$

$\xi_0 = 0.18 \frac{\hbar v_F}{k_B T_c}$ coherence length

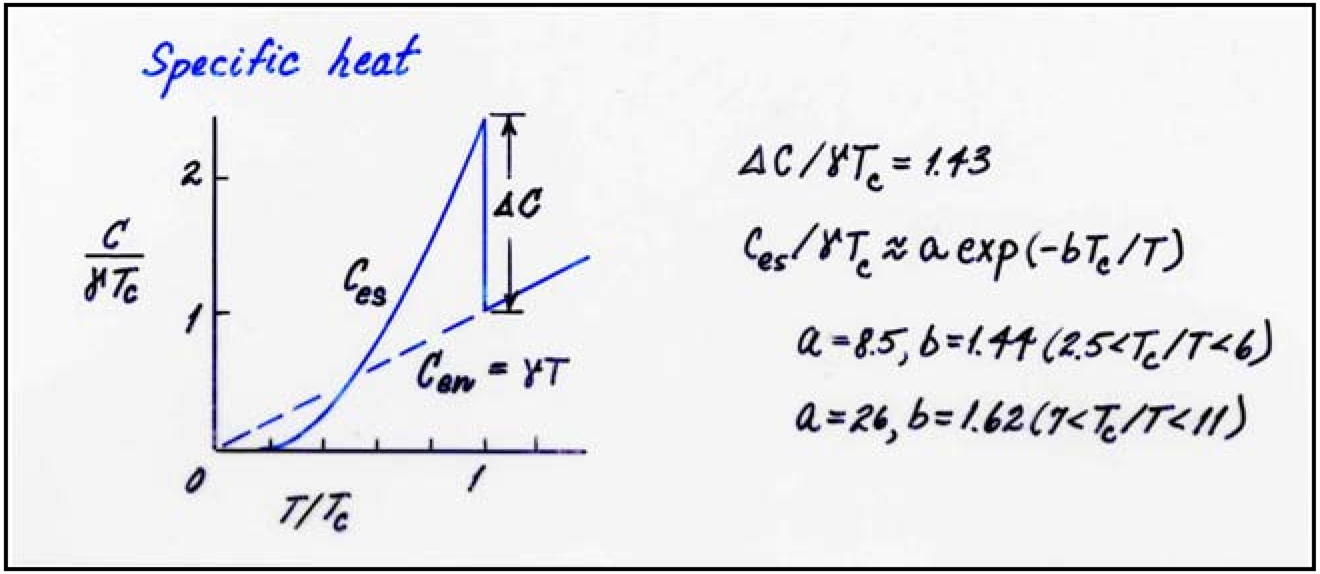
Energy gap (order parameter)

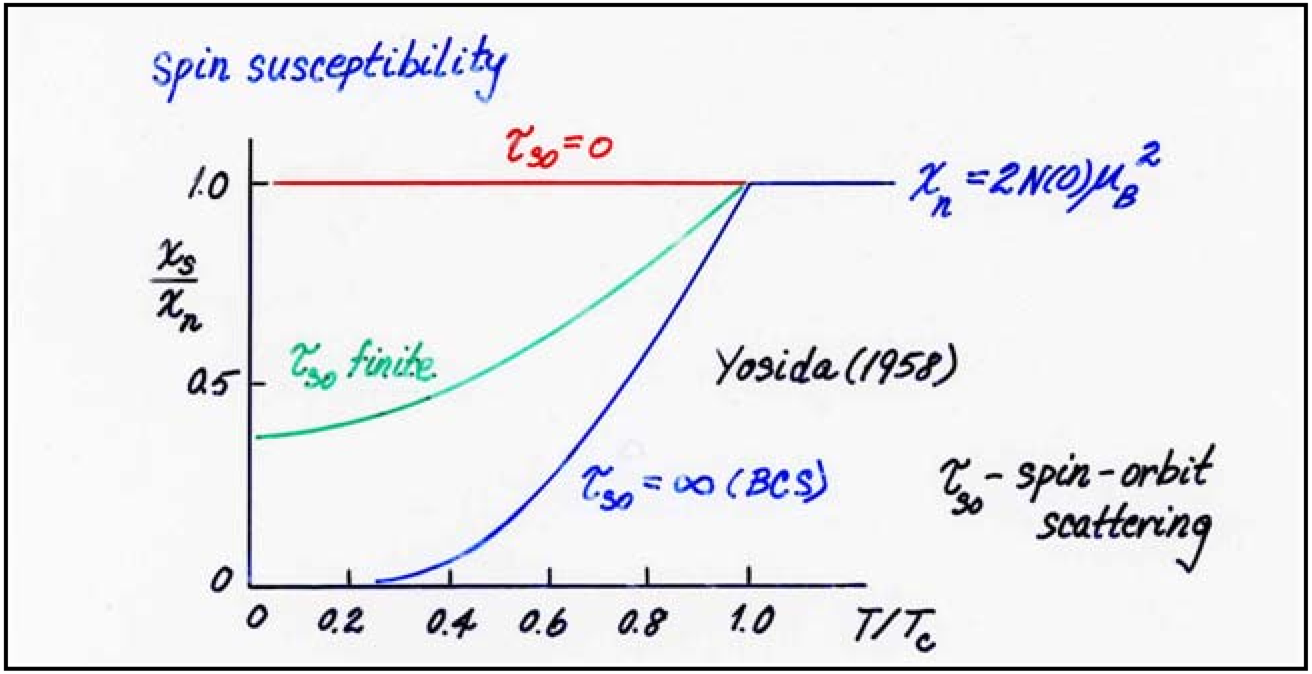


$$\frac{\Delta(T)}{\Delta(0)} \approx 1.74\left(1 - \frac{T}{T_c}\right)^{1/2}$$

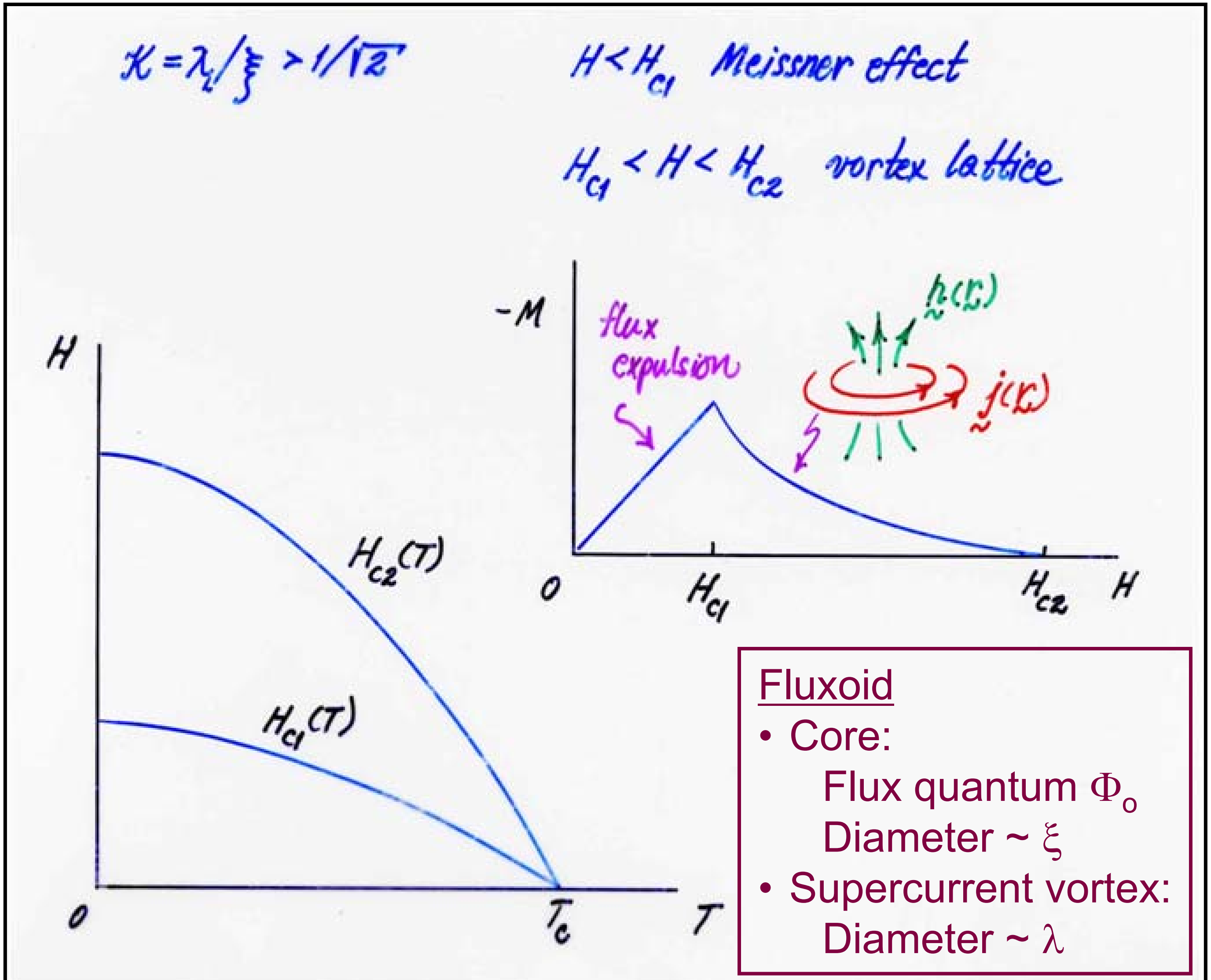
for $T \approx T_c$

Conventional superconductivity





Type II superconductivity (compounds, alloys)



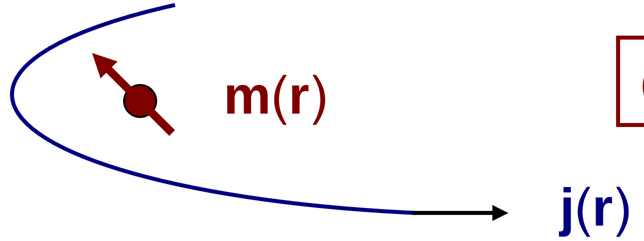
<u>Fluxoid</u>

- Core:
  - Flux quantum Φ_o
  - Diameter ~ ξ
- Supercurrent vortex:
  - Diameter ~ λ

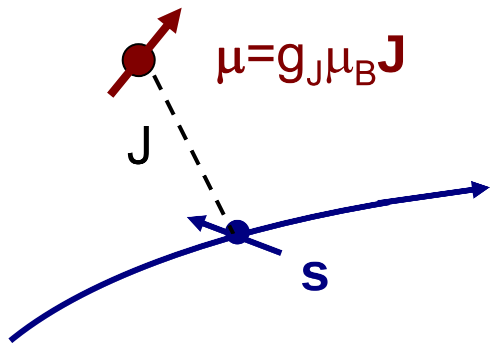
Superconducting-magnetic interactions

Superconductor containing R ions (magnetic moment μ)

Electromagnetic



$(e/mc)(\mathbf{p}\cdot\mathbf{A})$

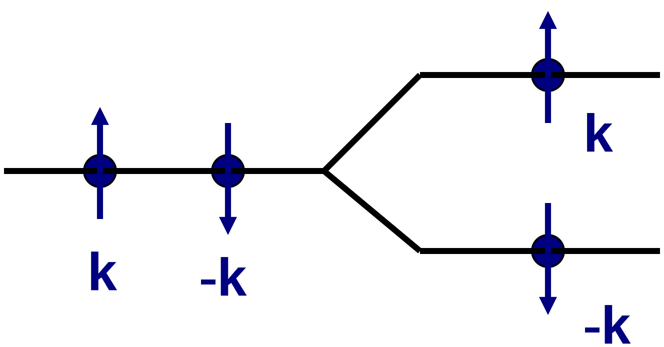


Exchange

$H_{ex} = -2J(g_J-1)\mathbf{J}\cdot\mathbf{s}$

"Pair breaking" interactions

Conventional SC: ($\mathbf{k}\uparrow$, $-\mathbf{k}\downarrow$)



Paramagnetic impurities in superconductors

Interplay between superconductivity and magnetism: History

- **Origin**

  Theory – *Ginzburg (57)*

  Experiments – *Matthias, Suhl, Corenzwit (58)*

- **Background (57-76)**

  Experiments

  - Binary and pseudobinary R impurity systems; e.g., $La_{1-x}R_x$, $Y_{1-x}R_xOs_2$
  - Rapid depression of T_c with x (x_{cr} ~ 1 at% for $La_{1-x}Gd_x$)
  - Results – provocative, inconclusive wrt coexistence of two phenomena (chemical clustering, short-range or “glassy” magnetic order)

  Theory

  - Striking predictions, inapplicable to systems then under investigation

  Spin-off

  - Understanding of effects of paramagnetic impurities on superconductivity – CEF, Kondo effect, LSF, etc.

- **Revival (~76)**

  Experiments – Binary, ternary and quaternary R and U compounds
  ⇒ new, unusual physical effects and phenomena

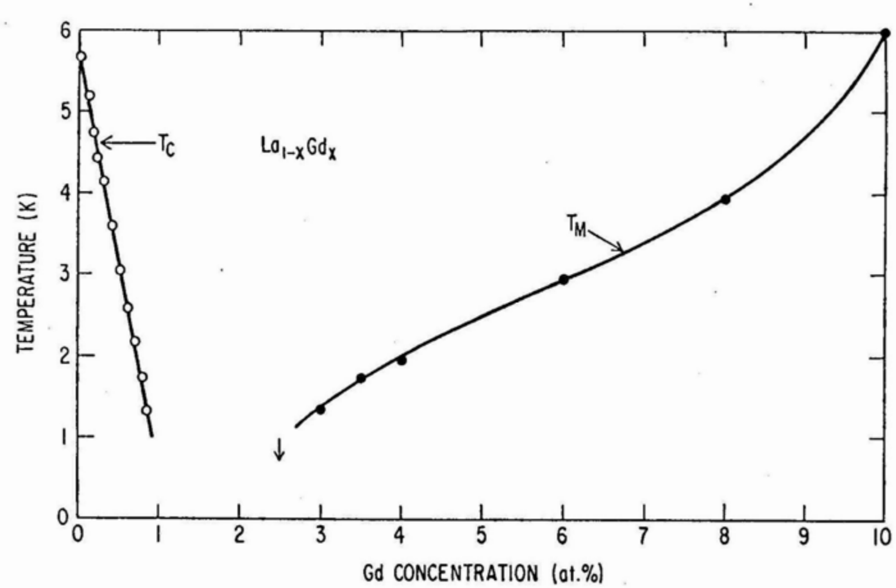
  Theory – Intense activity

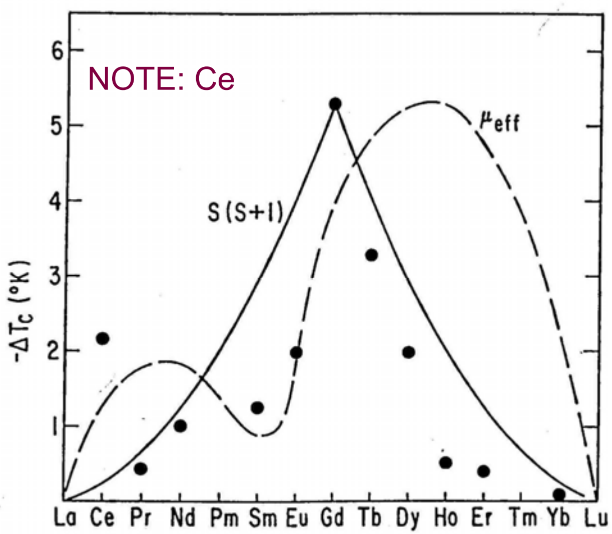
Paramagnetic impurities in superconductors

Matthias, Suhl, Corenzwit (58) —

- T_c (x) for $La_{1-x}R_x$
  R = Gd: $T_c(x) \to 0$ K for x ≈ 1 at.%

- Depression of T_c for x=1 at.%, $-\Delta T_c = T_{co} - T_c$, correlates with S of R solute



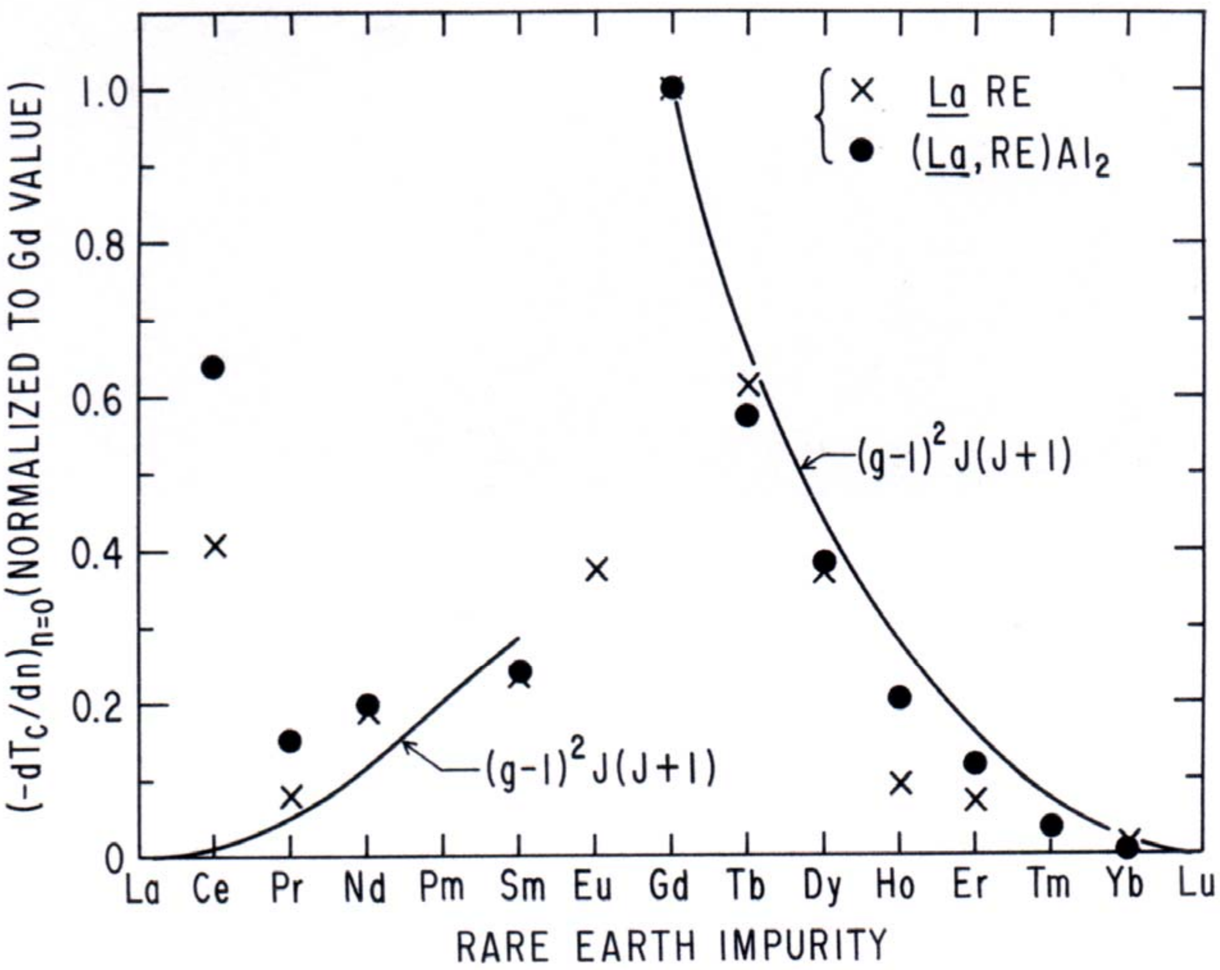


Herring (58); Suhl & Matthias (59) —

Exchange interaction: $H_{int} = -2(g_J-1)J\mathbf{J}\cdot s$

$\Delta T_c \propto J^2(g_J-1)^2J(J+1)$; deGennes factor $\equiv (g_J-1)^2J(J+1)$

Paramagnetic impurities in superconductors



LaRE (T_{co} = 6 K)

Matthias, Suhl, Corenzwit (58)

(LaRE)Al_2 (T_{co}= 3.3 K)

Maple (70)

J ~ 0.1 eV

Anomalous depression of T_c for Ce ⇒ hybridization of Ce localized 4f and itinerant electron states ⇒ $J \sim -\langle V_{kf}^2 \rangle / E_f < 0$ ⇒ Kondo effect

LaCe: *Sugawara, Eguchi (66)*; (LaCe)Al_2: *Maple, Fisk (68)*

Paramagnetic impurities in superconductors

Pair breaking ⇒ rapid suppression of SC

Two cases: J > 0 (ferromagnetic)

J < 0 (antiferromagnetic)

▸ $J > 0$ (ferromagnetic)

$T_c/T_{c_0} = Un(\alpha/\alpha_{cr})$ Abrikosov & Gor'kov (AG-1960)

α – pair breaking parameter

$$\alpha = \tau_{ex}^{-1} = \hbar^{-1} n N(0) J^2 (g_J - 1)^2 J(J+1)$$

$\alpha_{cr} = k_B T_{c_0} / 4\hbar\gamma$ ($\ln \gamma = 0.57721$ – Euler's const.)

Explicitly –

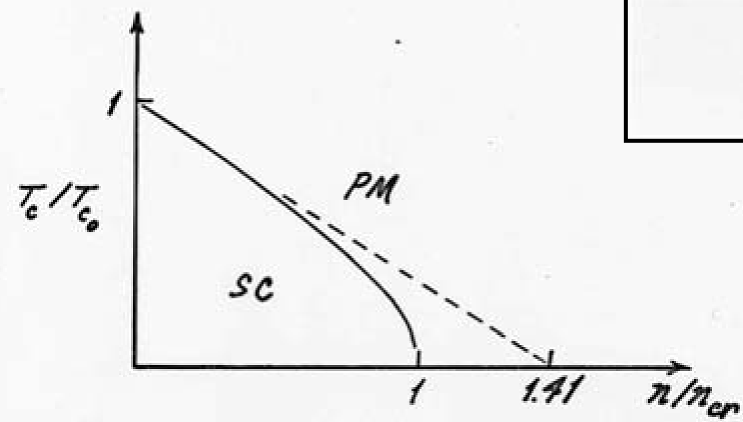
$$\ln(T_c/T_{c_0}) = \psi\left(\tfrac{1}{2}\right) - \psi\left(\tfrac{1}{2} + 0.14\,\frac{\alpha T_{c_0}}{\alpha_{cr} T_c}\right)$$

ψ – digamma function

Limit $\alpha \rightarrow 0$ –

$$T_c/T_{c_0} = 1 - 0.691(\alpha/\alpha_{cr}) = 1 - 0.691(n/n_{cr})$$

Paramagnetic impurities in superconductors



Linear region $n/n_{cr} \ll 1$

$$\left.\frac{dT_c}{dn}\right|_{n=0} = -(\pi^2/2)\,k_B^{-1}\,N(0)\,\mathcal{J}^2 \underbrace{(g_J-1)^2 J(J+1)}_{\text{de Genne's factor (maximum at Gd)}}$$

Other predictions –

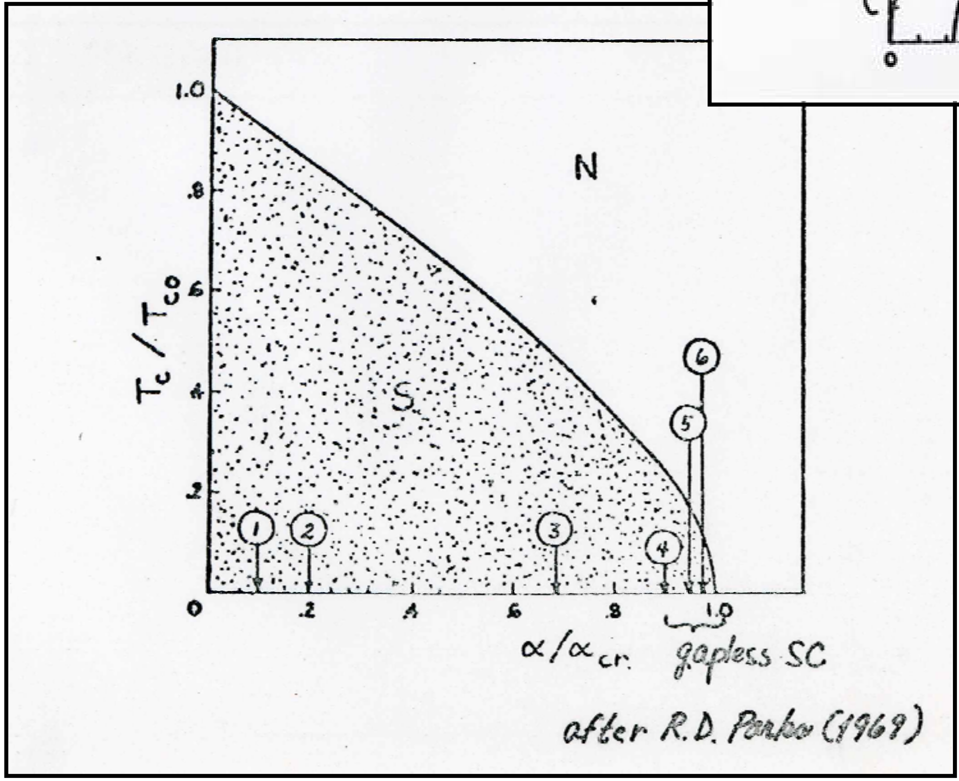
$\Delta C/\Delta C_0 = Vn\,(T_c/T_{c_0})$.

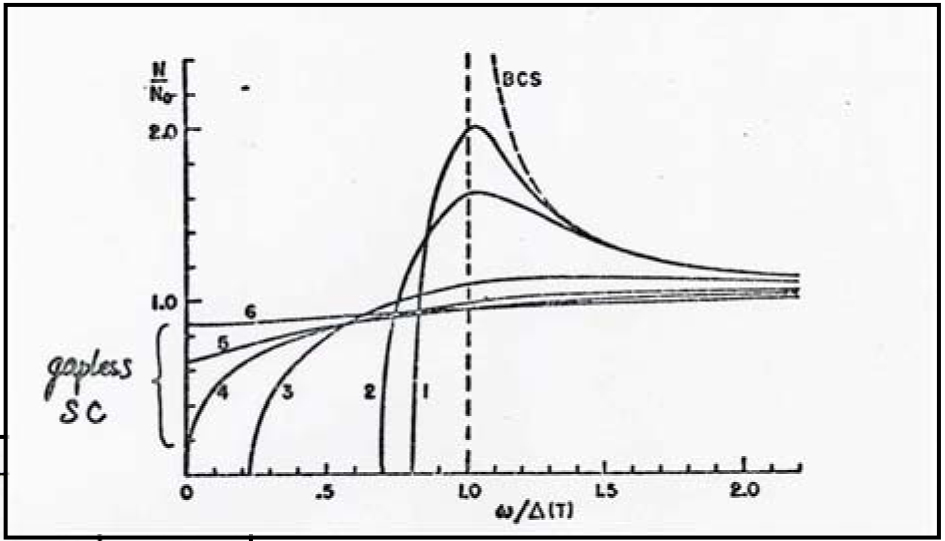
deviates from BCS law of corresponding states ($\Delta C/\Delta C_0 = T_c/T_{c_0}$)

Gapless SC $\quad \Delta \not\leftrightarrow \Omega$

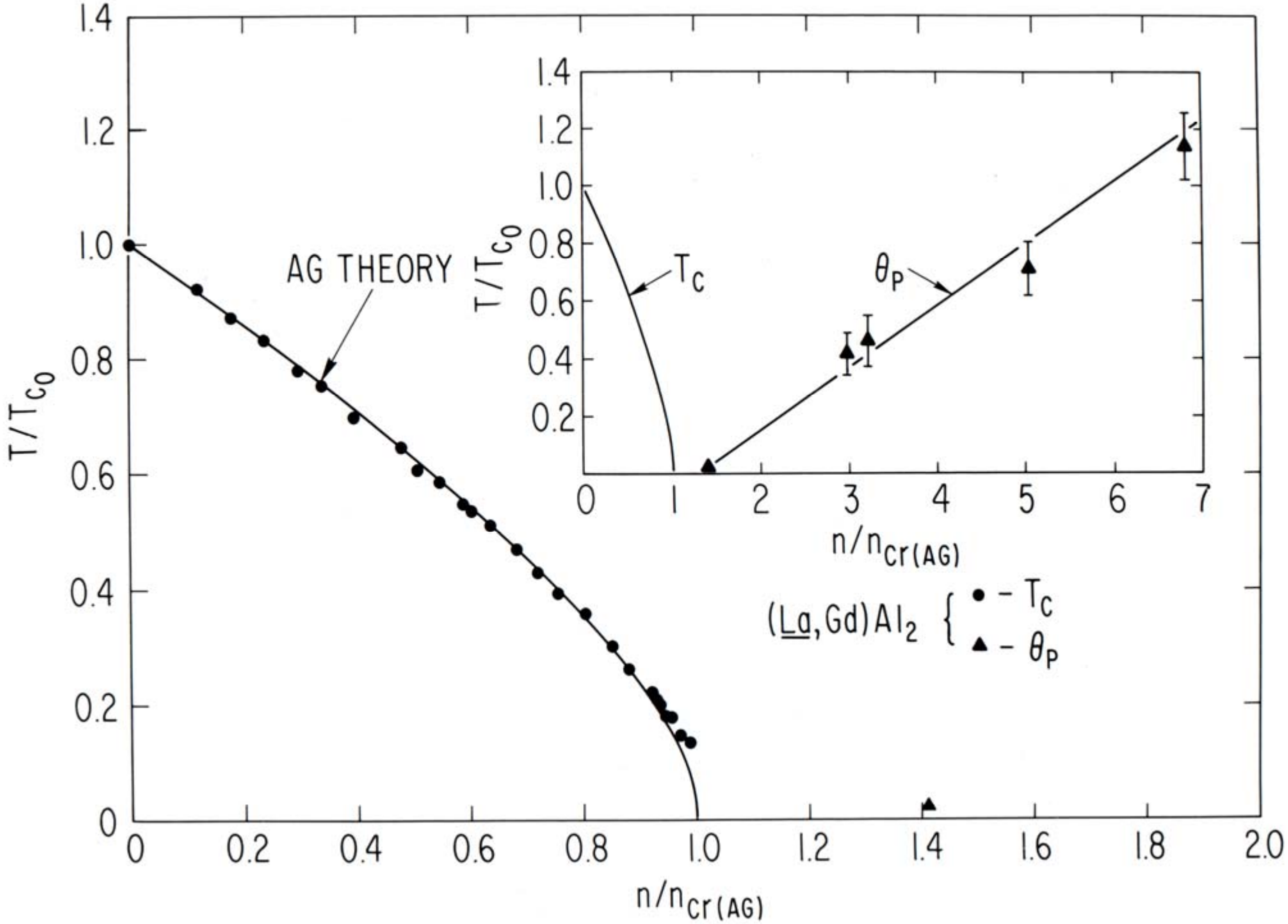
$\Omega \to 0$ faster than $\Delta \to 0$ with α

Gapless superconductivity





$La_{1-x}Gd_xAl_2$; T_c vs x phase boundary

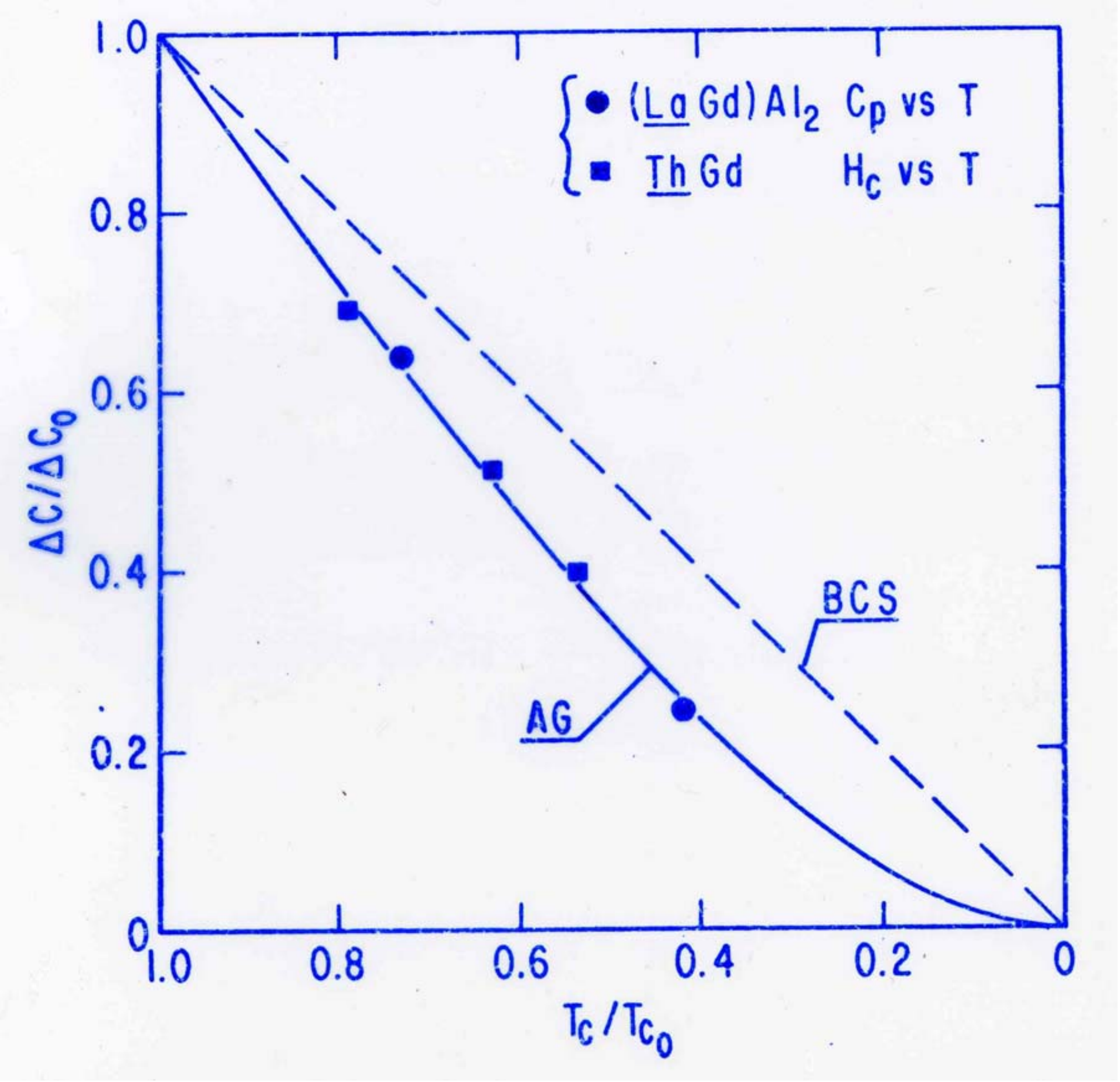


T_{co} = 3.3 K
n_{cr} = 0.59 at.% Gd

"Gapless" superconductivity
Woolf, Reif (65) - tunneling $Pb_{1-x}Gd_x$
Finnemore et al. (65) - specific heat $La_{1-x}Gd_x$

M. B. Maple (68)

$La_{1-x}Gd_xAl_2$; specific heat jump vs T_c



W. R. Decker, D. K. Finnemore (68)
C. A. Luengo, M. B. Maple (73)

SCing metal containing paramagnetic impurities (spin S)

- AFM exchange interaction

  $H_{ex} = -2J\mathbf{S}\cdot\mathbf{s}$ with $J < 0$

- Formation of many body singlet state below T_K

  $T_K \sim T_F \exp(-1/N(E_F)|J|)$ $T_K \rightarrow$ effective T_F

- <u>Normal State</u>
  - $T > T_K$: Local moment behavior

    $\chi(T) \sim N\mu_{eff}^2/3k_B(T-\theta)$ where $\theta \sim -3T_K$

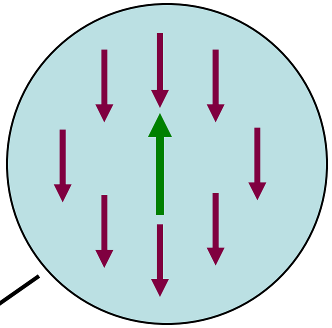
    $\rho(T) \sim -\ln T$ ("resistivity minimum")

  - $T \ll T_K$: Many body singlet

    Nonmagnetic heavy Fermi liquid (FL)

    $\chi(T) \propto \gamma(T) = C(T)/T \sim$ const.

    $\rho(T) \approx \rho(0)[1-(T/T_K)^2]$



- Superconducting state

  Competition between:
  (1) Singlet spin paired (k↑,-k↓) SCing state ($E_{SC} \sim k_B T_c$);
  (2) Kondo many body singlet state ($E_K \sim k_B T_K$)

  - $T_K << T_{co}$: Reentrant $T_c(x)$ curve!
  - $T_K >> T_{co}$: Exponential-like depression of T_c with x
  - $T_K \approx T_{co}$: Maximum in initial rate of depression of T_c

  Theory: *Müller-Hartmann, Zittartz (70-71);*
  Zuckermann (68): Ludwig, Zuckermann (71)

Paramagnetic impurities in superconductors

$$T_c / T_{c_0} = \mathcal{U}n\,(\alpha/\alpha_{cr})$$

$$\alpha/\alpha_{cr} = n\,B\left\{\frac{\pi^2 S(S+1)}{\ln^2(T/T_K) + \pi^2 S(S+1)}\right\}$$

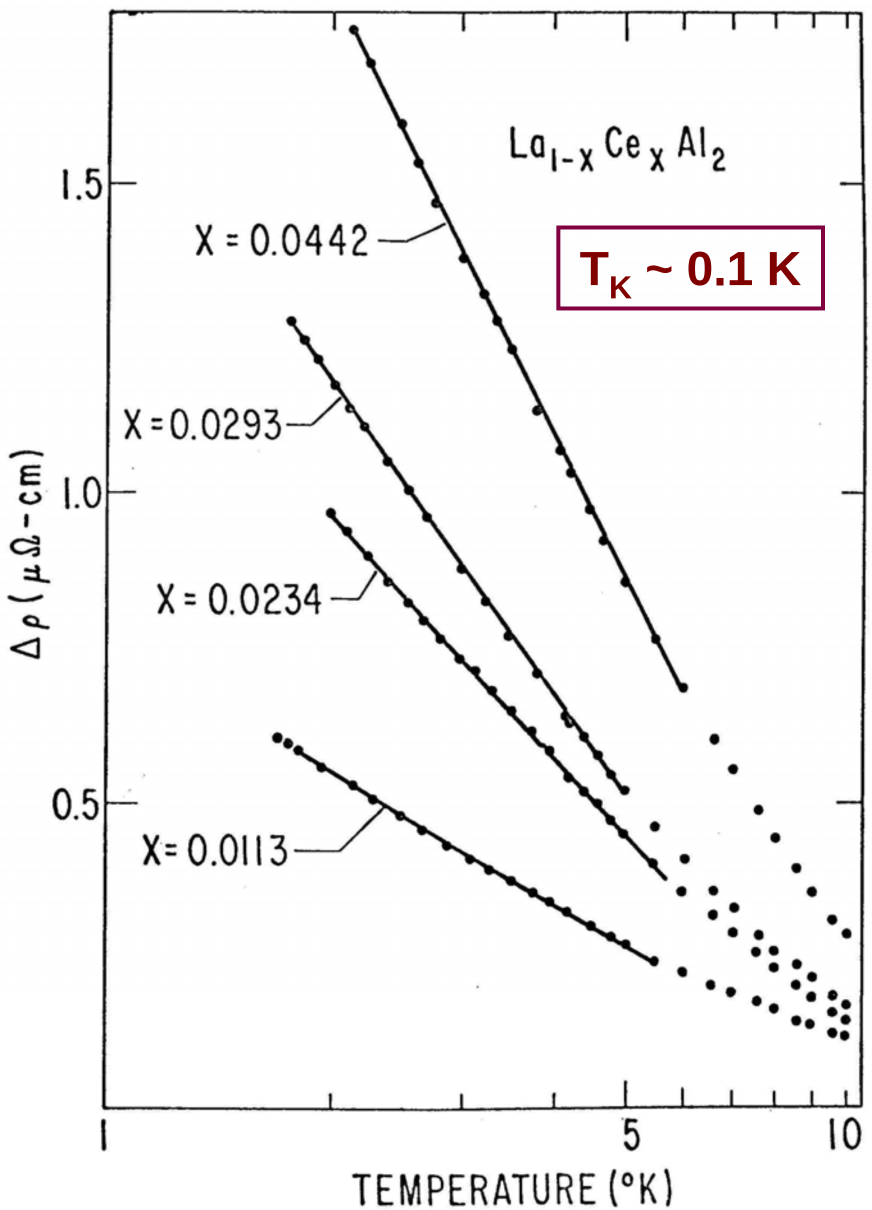
Müller-Hartmann & Zittartz (MHZ - 1971)

Other predictions -

$\Delta C/\Delta C_0$ vs T_c/T_{c_0} deviates from both BCS law of corresponding states & AG theory

Bound state in energy gap

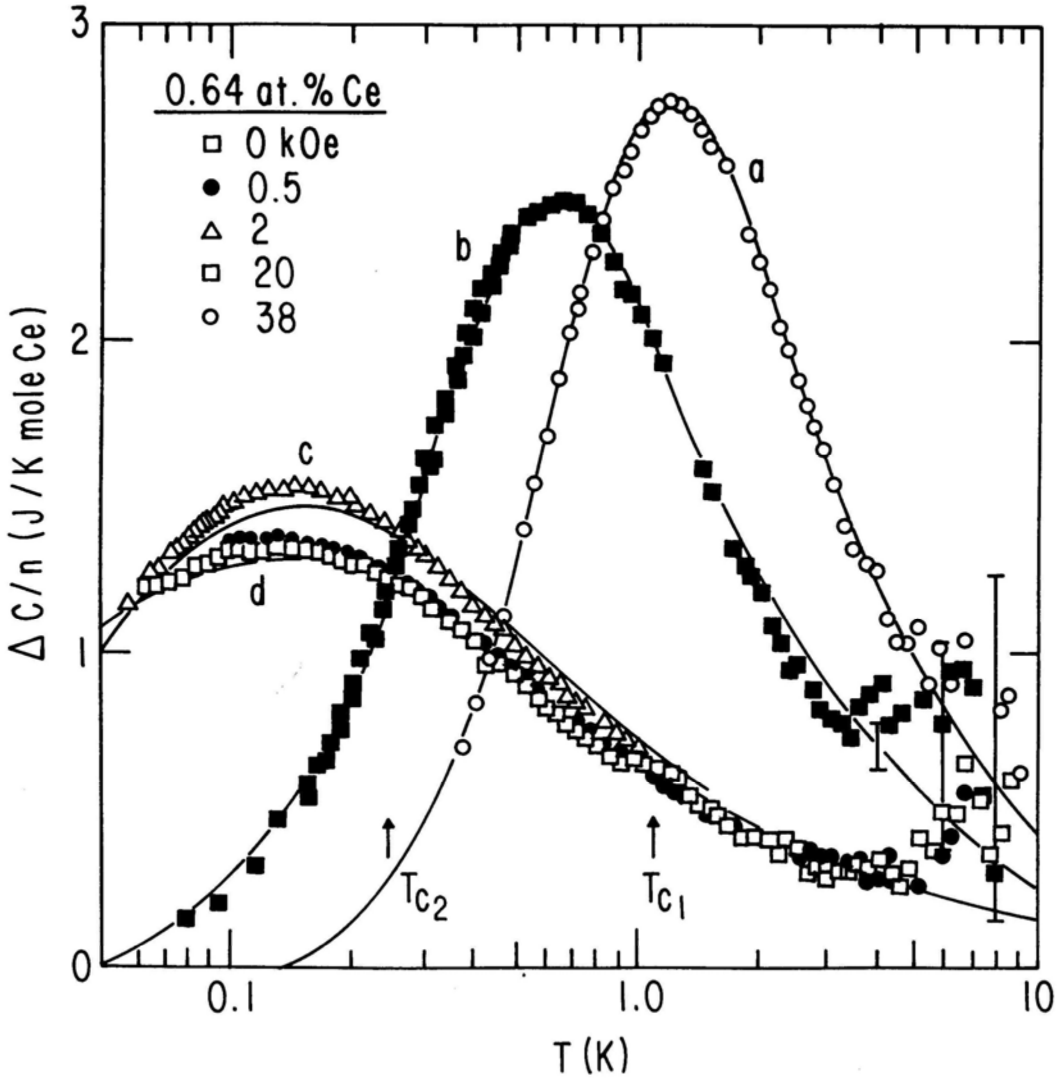
Kondo effect in $La_{1-x}Ce_xAl_2$: electrical resistivity



Magnetic scattering
"Kondo" contribution
to electrical resistivity:
$\Delta\rho(T) = \rho(x,T) - \rho(0,T)$

Maple, Fisk (68)

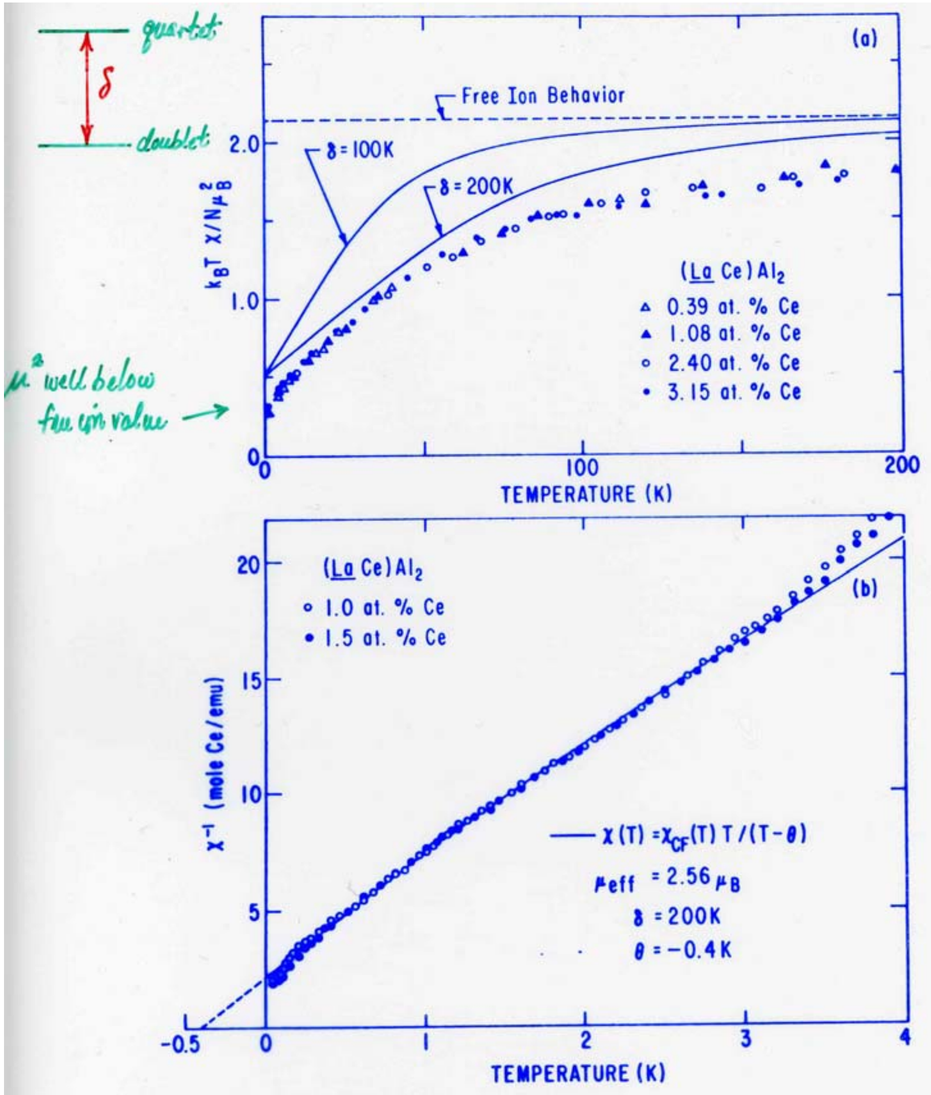
Kondo effect in $La_{1-x}Ce_xAl_2$: specific heat



Curve d - theory of Bloomfield and Hamann for S = 1/2 and T_K = 0.42 K (H = 0)

S. D. Bader, N. E. Phillips, M. B. Maple, C. A. Luengo (75)

Kondo effect in $La_{1-x}Ce_xAl_2$: magnetic susceptibility

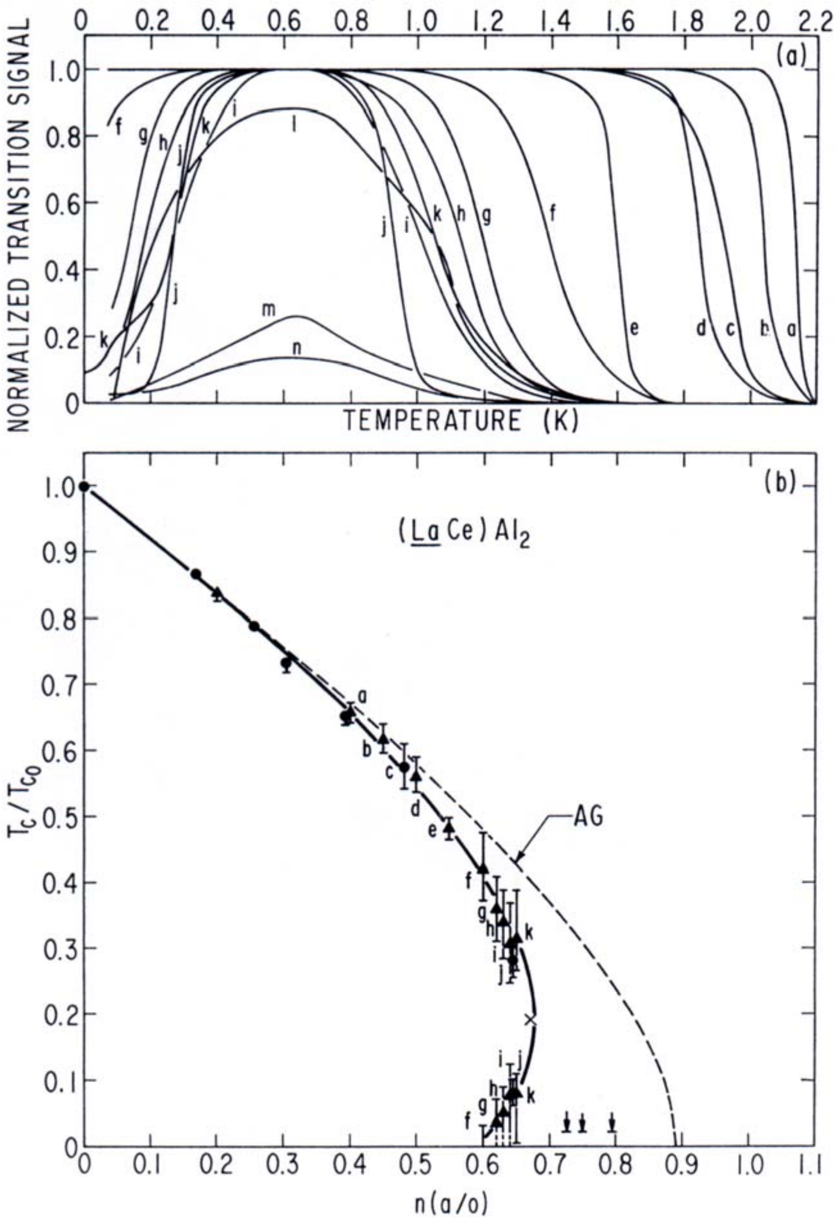


- Ce has ground state doublet and excited state quartet with splitting δ ~ 100 K in CEF

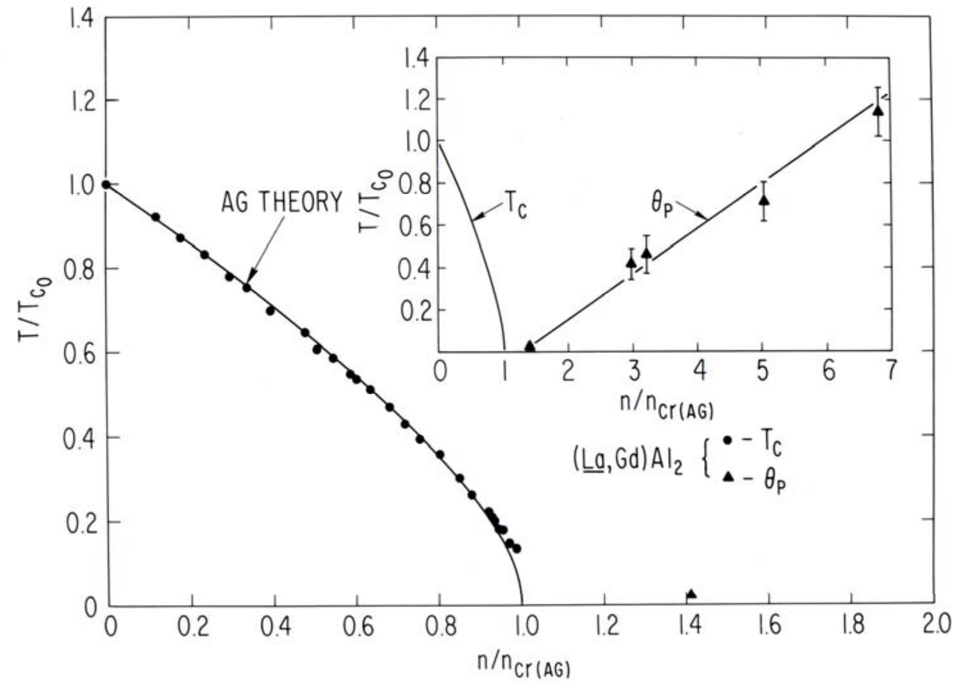
□ $\theta_p \approx -0.5$ K $\Rightarrow$ $T_K \approx 0.1$ K

(a) M. B. Maple (69)
(b) W. Felsch, K. Winzer, G. v. Minnigerode (75)

Kondo effect in $La_{1-x}Ce_xAl_2$: reentrant T_c vs x curve

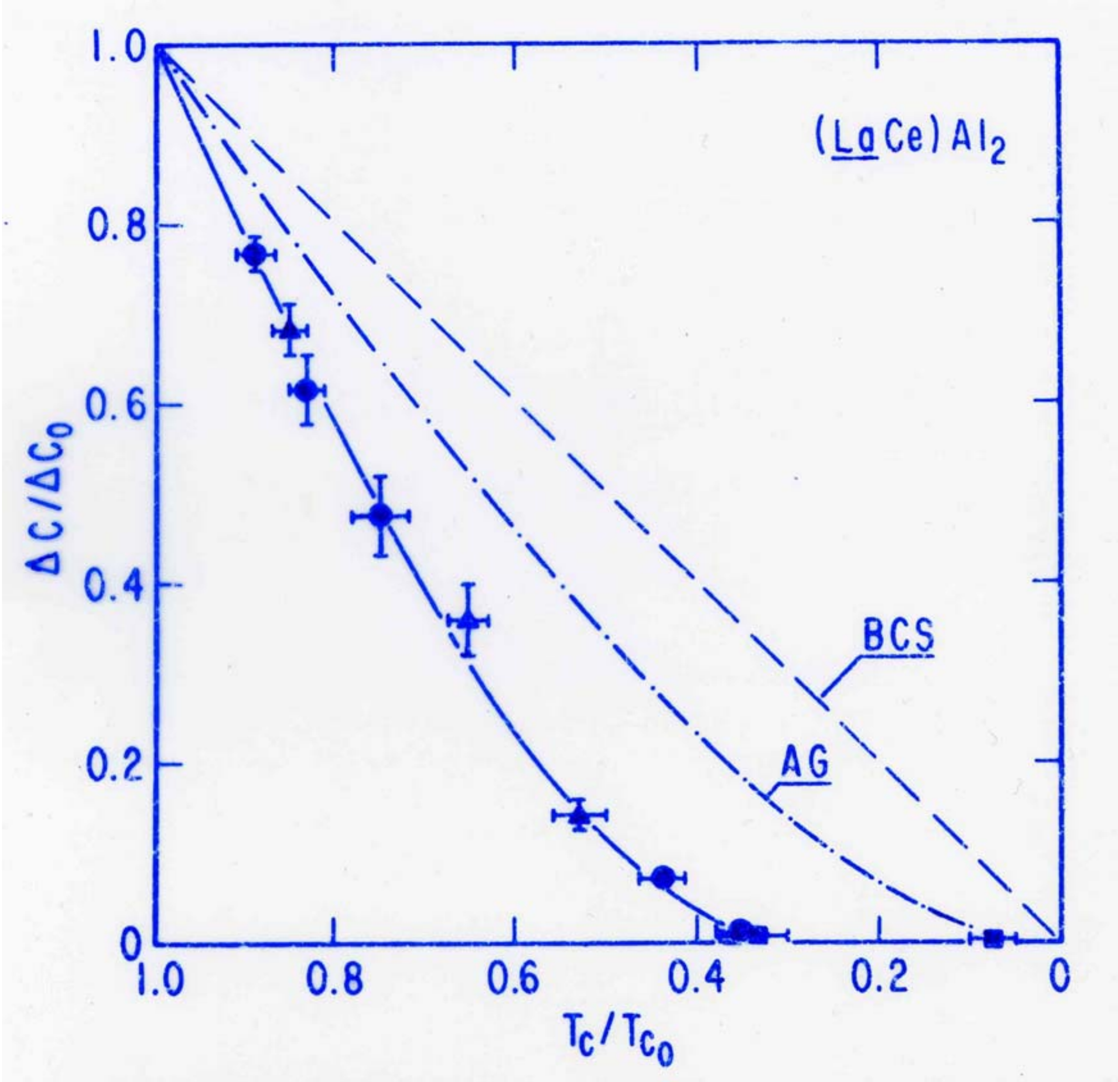


- Kondo effect: $T_K << T_{co}$
  "reentrant SC"
- *Riblet, Winzer (71) (U. Köln)*
- *Maple, Fertig, Mota, DeLong, Wohlleben, Fitzgerald (72) (UCSD)*



Maple (68)

Kondo effect in $La_{1-x}Ce_xAl_2$: specific heat jump vs T_c



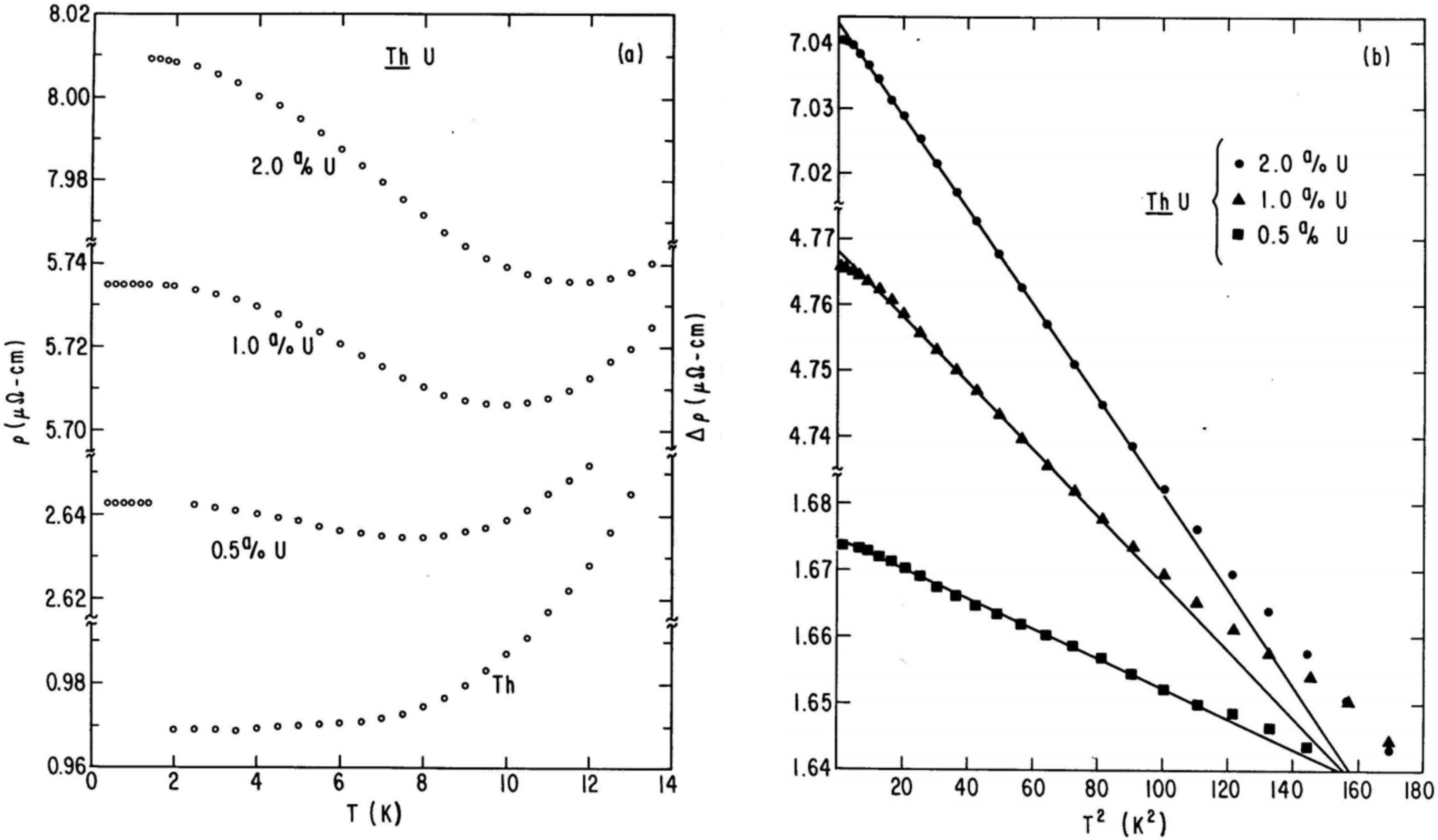
- *C. A. Luengo, M. B. Maple, W. A. Fertig (72)*
- *H. Armbrüster, F. Steglich (73)*
- *S. D. Bader, N. E. Phillips, M. B. Maple, C. A. Luengo (73)*

Kondo effect in $Th_{1-x}U_x$: electrical resistivity

- $Th_{1-x}U_x$: conventional Kondo effect (Fermi liquid - low T)

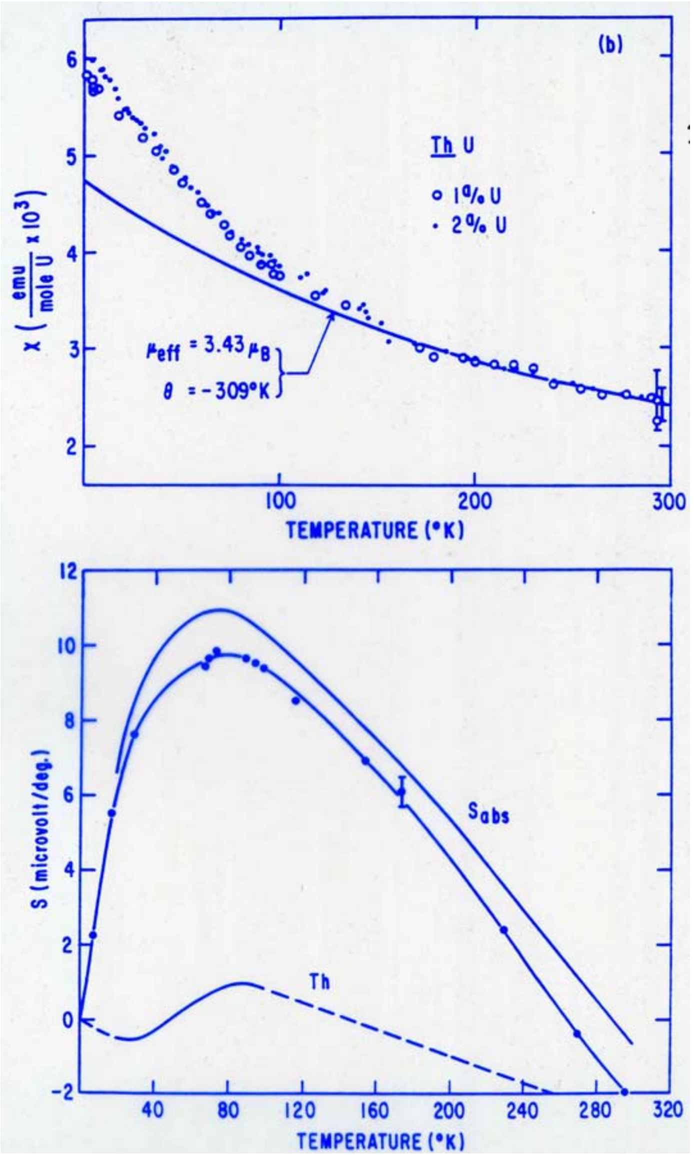
□ $\Delta\gamma \approx 270$ mJ/mol U-K^2

□ $\Delta\rho(T) = \rho_o[1-(T/T_K)^2]$; $T_K \approx 100$ K



M. B. Maple et al. (70)

Kondo effect in $Th_{1-x}U_x$: magnetic susceptibility and thermopower



$Th_{1-x}U_x$:

□ $\chi(T) = C/(T-\theta)$

□ $\theta \approx -3\ T_K$

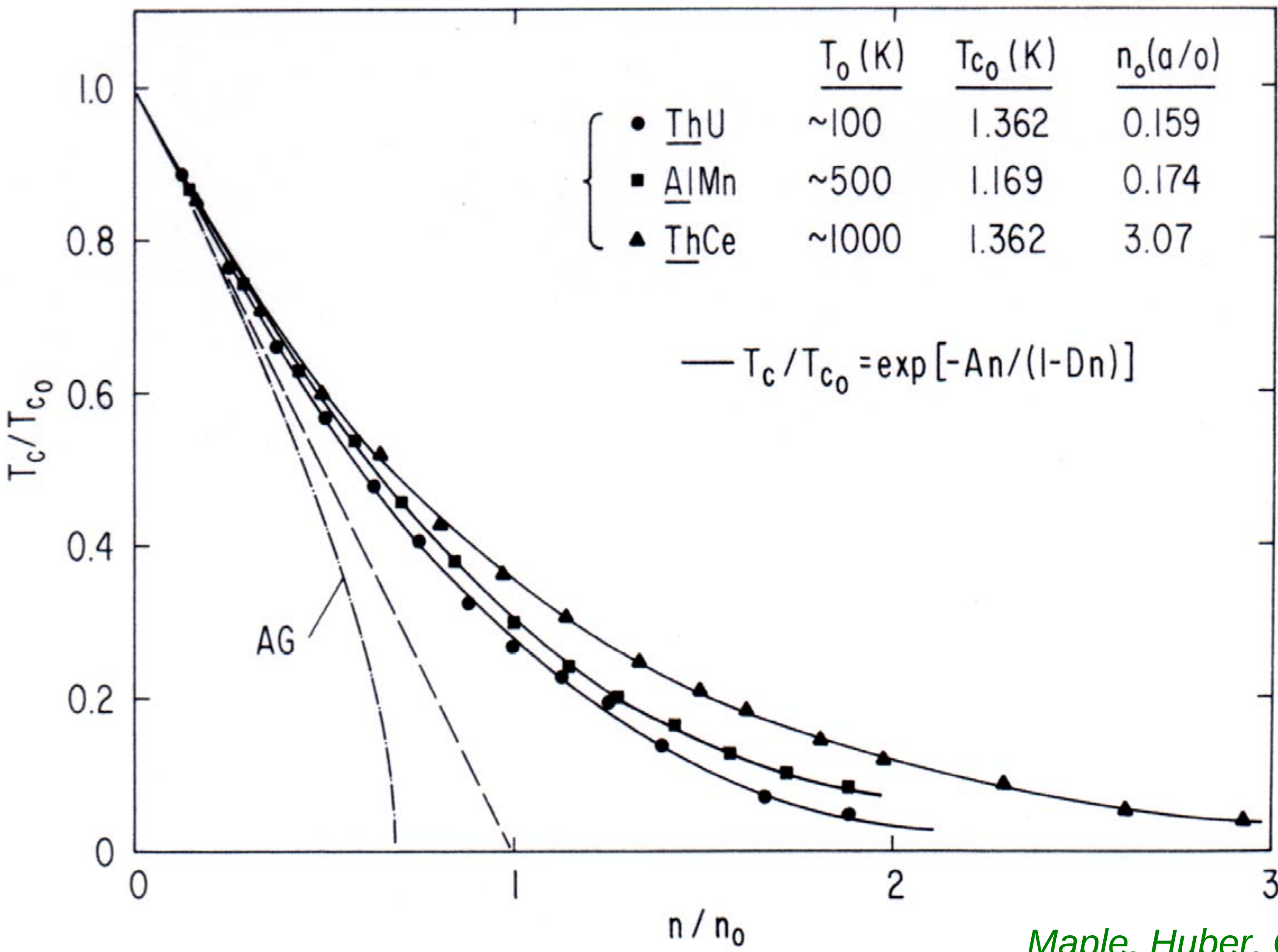
- $C = N\mu_{eff}^2/3k_B$

- Peak in thermoelectric power
  $\Rightarrow T_K \approx 100$ K

M. B. Maple et al. (70)

Kondo effect in $Th_{1-x}U_x$: exponential T_c vs x curve

Comparison of T_c vs x curves of $Th_{1-x}U_x$, $Al_{1-x}Mn_x$, $Th_{1-x}Ce_x$



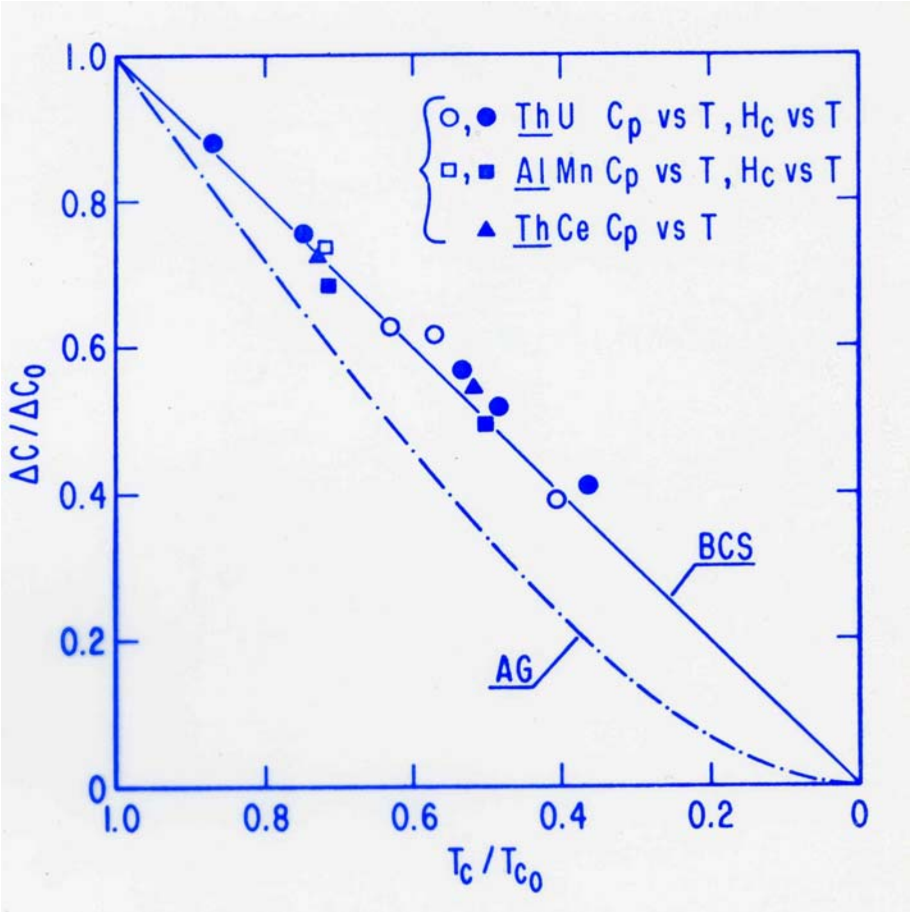
Kondo effect:

$T_K >> T_{co}$

Maple, Huber, Coles, Lawson (70); Huber, Maple (70)

Kondo effect in $Th_{1-x}U_x$: specific heat jump vs T_c

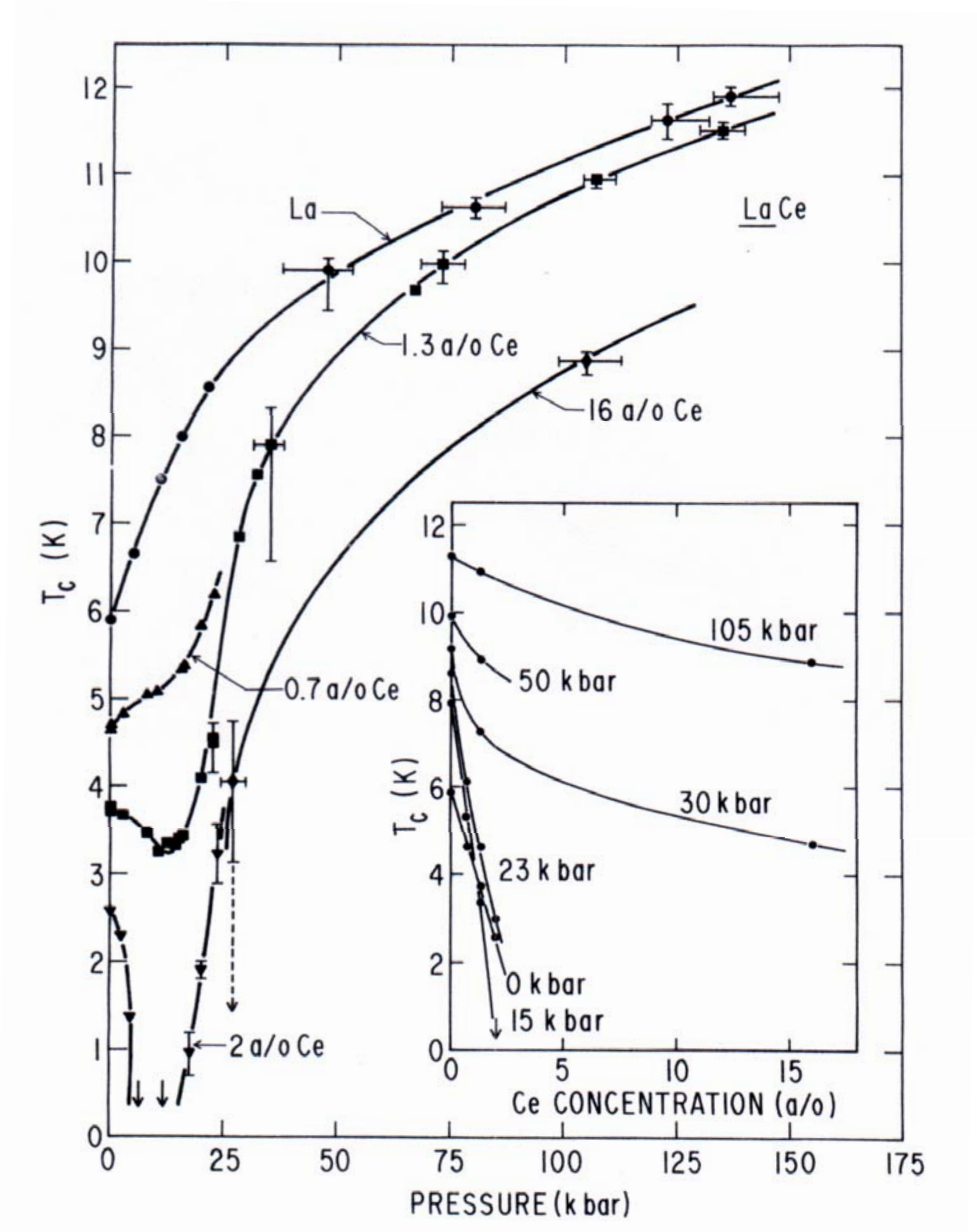
Comparison of $\Delta C/\Delta C_o$ vs T_c/T_{co} curves of $Th_{1-x}U_x$, $Al_{1-x}Mn_x$, $Th_{1-x}Ce_x$



$T_c << T_K$: Conforms to BCS law of corresponding states $\Delta C/\Delta C_o = T_c/T_{co}$

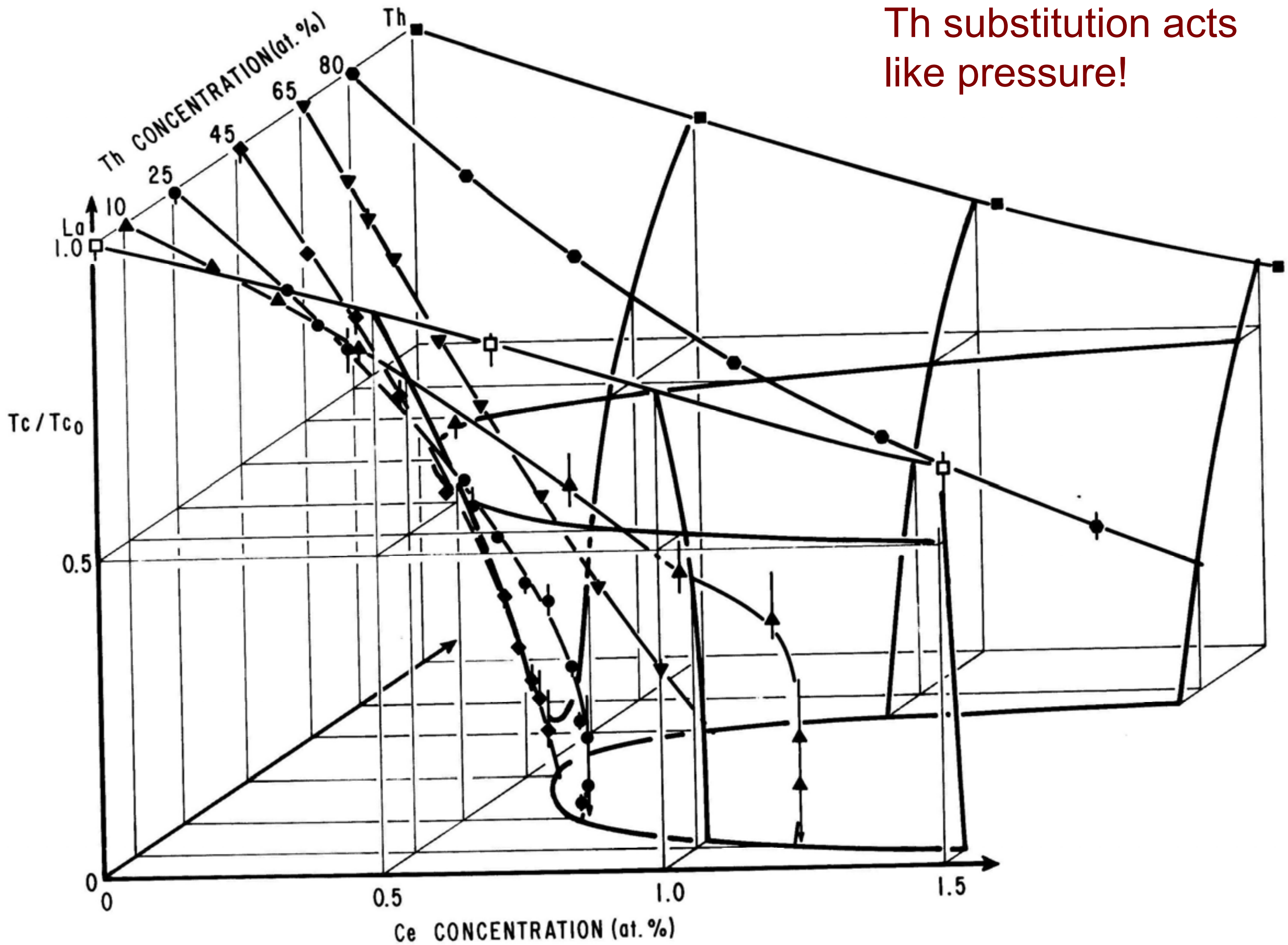
- (○) C. A. Luengo, J. M. Cotignola, J. Sereni, A. R. Sweedler, M. B. Maple, J. G. Huber (72)
- (●) H. L. Watson, D. T. Peterson, D. K. Finnemore (73)
- (□) D. L. Martin (61)
- (■) F. W. Smith (72)
- (▲) C. W. Dempesy (70)

Pressure-induced demagnetization of Ce impurities in La



- |J| & T_K increase with P
  $\Rightarrow$ T_K/T_{co} increases with P from << 1 to >> 1 through $T_K/T_{co} \approx 1$ at ~15 kbar
  $\Rightarrow$ maximum in ΔT_c at ~15 kbar
- Analogue of Ce γ-$\propto$ transition
- *Maple, Wittig, Kim (69)*

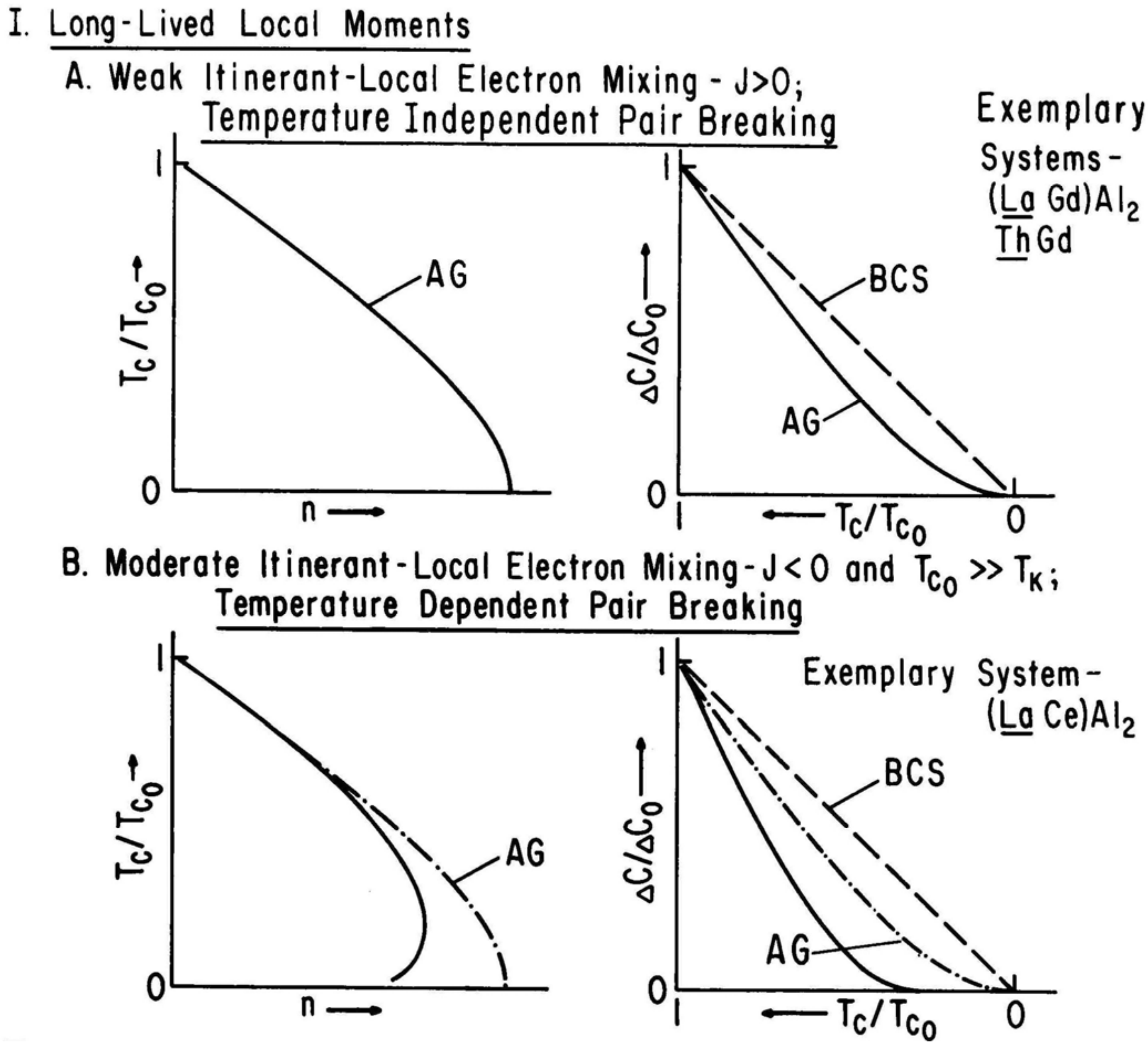
Demagnetization of Ce impurities in $(La_{1-y}Th_y)_{1-x}Ce_x$



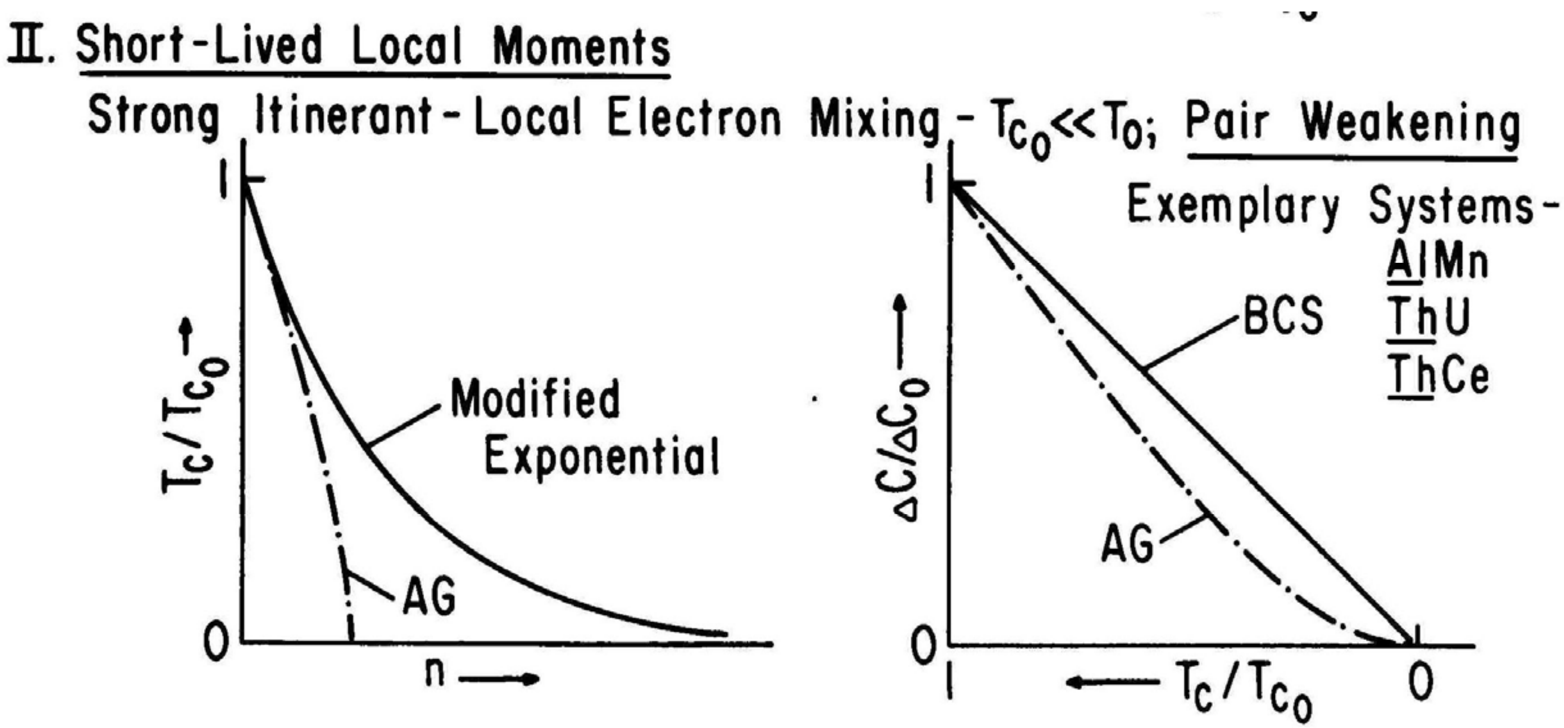
Th substitution acts like pressure!

Huber, Fertig, Maple (72)

Paramagnetic impurities in superconductors: summary



Paramagnetic impurities in superconductors: summary



Magnetic field induced superconductivity (MFIS)

Origin of the upper critical field H_{c2}

(1) $\frac{e}{mc}(\underset{\sim}{p}\cdot\underset{\sim}{A}) = \frac{e\hbar}{mc}(\underset{\sim}{k}\cdot\underset{\sim}{A}) \longrightarrow H_{c2}^{*}$

(2) $-\underset{\sim}{\mu}\cdot\underset{\sim}{H} = -g\mu_B(\underset{\sim}{s}\cdot\underset{\sim}{H}) \longrightarrow H_p$

(1) ORBITAL CRITICAL FIELD H_{c2}^{*}

$$H_{c2}^{*} = \Phi_0/2\pi\xi^2 \qquad \Phi_0 = hc/2e$$

(2) PARAMAGNETIC LIMITING FIELD H_p

normal state

$$F_n(H) = F_n(0) - \frac{1}{2}\chi_n H^2$$

superconducting state

$$F_s(H) = F_s(0) - \frac{1}{2}\chi_s H^2$$

first order transition from superconducting to normal state when

$$F_n(H_p) - F_s(H_p) = 0$$

$$= [F_n(0) - F_s(0)] - \frac{1}{2}(\chi_n - \chi_s) H_p^2$$

$$= \frac{1}{2}N(0)\Delta^2 - \frac{1}{2}(\chi_n - \chi_s) H_p^2$$

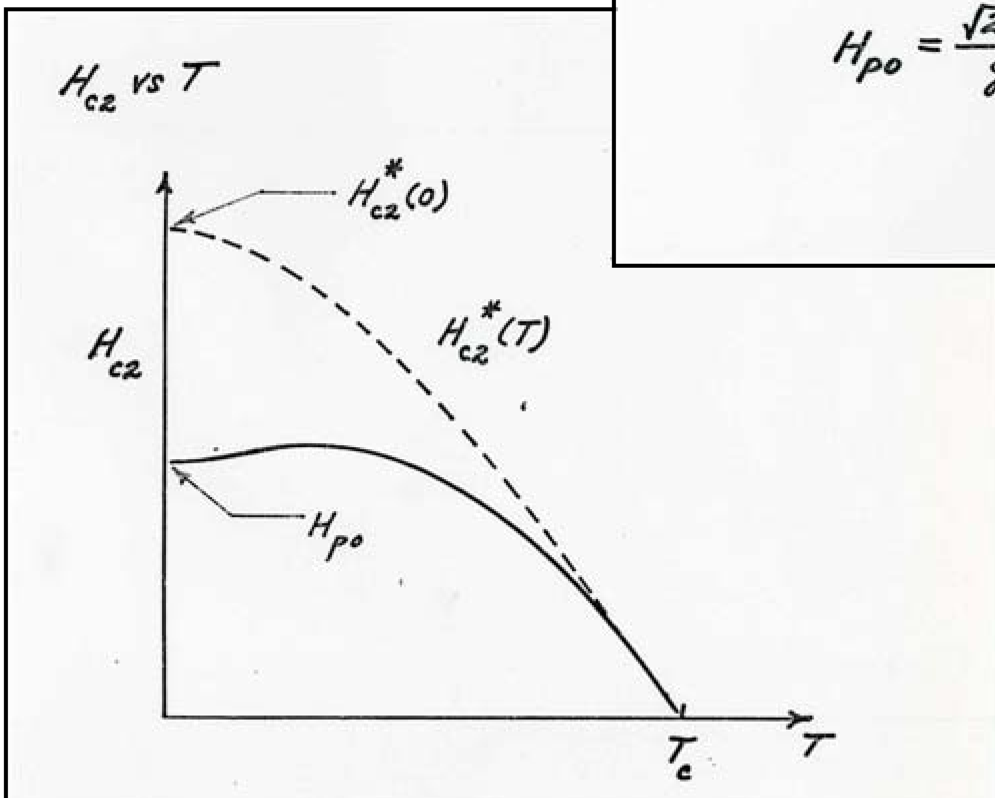
Origin of the upper critical field H_{c2}

$$\Rightarrow \quad H_p = \left[\frac{N(0)}{\chi_n - \chi_s}\right]^{1/2} \cdot \Delta$$

BCS superconductor at $T=0$ $\begin{cases} \chi_s = 0 \\ \chi_n = \frac{1}{2} N(0) g^2 \mu_B^2 \end{cases}$

$$H_{p0} = \frac{\sqrt{2}\,\Delta(0)}{g\mu_B} = 18.4\, T_c \; (kOe)$$

Clogston - Chandrasekhar limit (1962)



To increase H_{c2} above H_p

- Increase spin-orbit scattering $\Rightarrow$ increase χ_s
- Compensate applied field with exchange field $\Rightarrow$ MFIS

SCHEME FOR RAISING PARAMAGNETICALLY LIMITED H_{c2}

Consider

$(Eu_{1-x}M_x)Mo_6S_8$ where M = Sn, Pb, La, Ho, Yb

H_{c2} determined by two interactions which "break" superconducting electron pairs $(\underset{\sim}{k}\uparrow;\underset{\sim}{k}\downarrow)$:

(1) $\underset{\sim}{p}\cdot\underset{\sim}{A} = \hbar(\underset{\sim}{k}\cdot\underset{\sim}{A}) \rightarrow$ orbital critical field H_{c2}^*

(2) $-\underset{\sim}{\mu}\cdot\underset{\sim}{H} = -2\mu_B(\underset{\sim}{s}\cdot\underset{\sim}{H}) \rightarrow$ paramagnetic limiting field H_p

Assume $H_{c2} \sim H_p < H_{c2}^*$

$Eu^{2+} \Rightarrow J = S = 7/2 \Rightarrow \mu = 7\mu_B$

$\mathcal{H}_{ex} = -2\mathcal{J}\,\underset{\sim}{S}\cdot\underset{\sim}{s} = -\underset{\sim}{\mu}\cdot\underset{\sim}{H}_{ex}{}'$ where $\mu = 2\mu_B\underset{\sim}{s}$, $\underset{\sim}{H}_{ex}{}' = (\mathcal{J}/\mu_B)\underset{\sim}{S}$

$\underset{\sim}{H}_T = \underset{\sim}{H} + \underset{\sim}{H}_{ex}$ where $\underset{\sim}{H}_{ex} = n\underset{\sim}{H}_{ex}{}'$

$\mathcal{J} < 0 \Rightarrow H_T = H - |H_{ex}| < H$

$H_{ex} \approx \langle S_z \rangle = SB_{7/2}(7\mu_B H/k_B T)$ (Brillouin function)

Magnetic field induced superconductivity

SC for $|H_T| = |H - |H_{ex}|| \le H_p$ or $|H_{ex}| - H_p \le H \le H_{ex} + H_p$

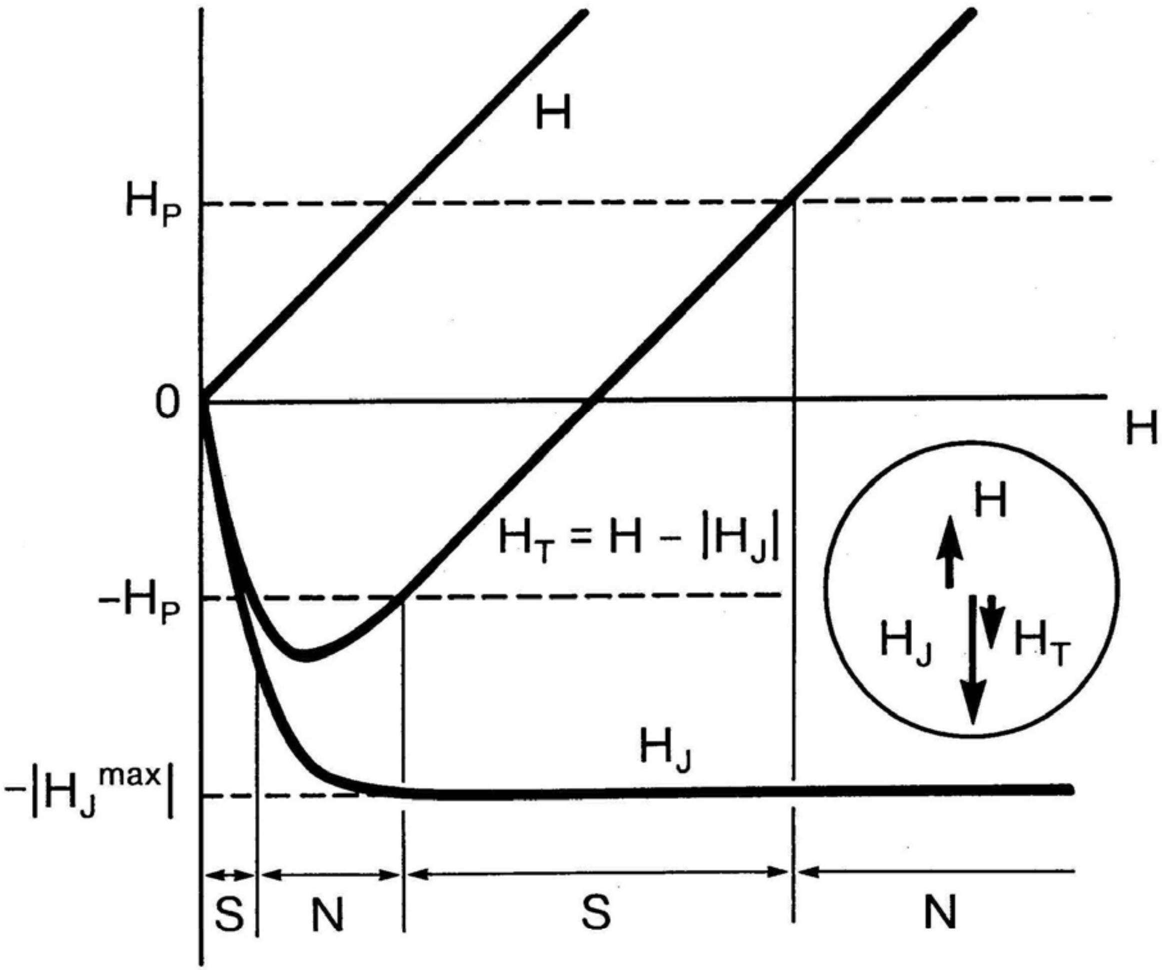
$\Rightarrow$ (1) Enhancement of H_{c2};

(2) Magnetic field induced SC!

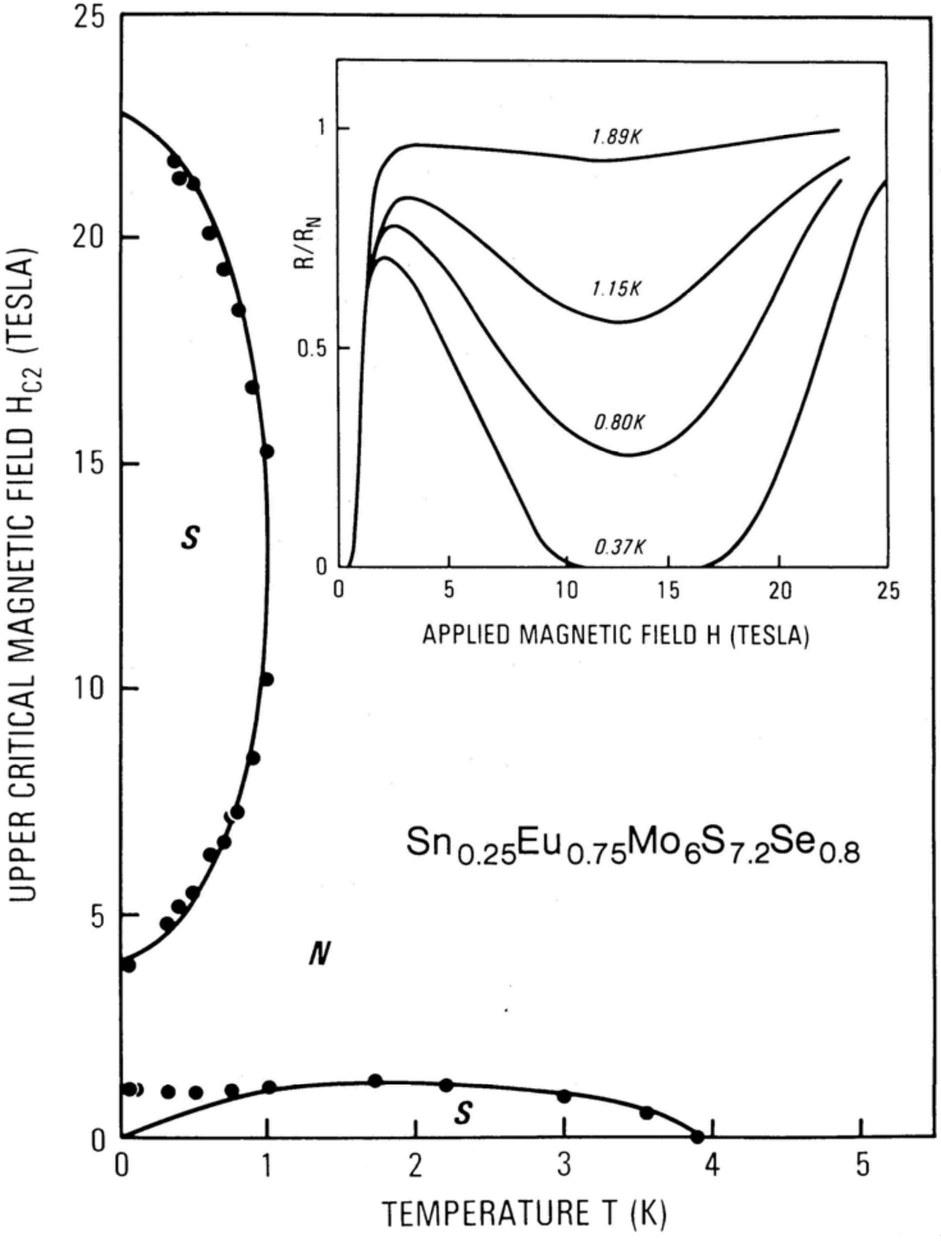
FM with $\mathcal{J} < 0$ in H $\rightarrow$ SC! (Jaccarino & Peter,'62)

Not yet been observed!

Magnetic field induced superconductivity



Magnetic field induced superconductivity



ϕ. Fisher et al. (83)

Magnetic ordering via RKKY interaction

$$\mathcal{H}_{int} = -2\mathcal{J}(g_J-1)\,\mathbf{J}\cdot\mathbf{s}\,\delta(\mathbf{r})$$

$$= -[2(g_J-1)\mathcal{J}/g\mu_B]\,\mathbf{J}\,\delta(\mathbf{r})\cdot(g\mu_B\mathbf{s})$$

$$= -\mathbf{H}_{eff}(\mathbf{r})\cdot\mathbf{m}$$

$$\mathbf{H}_{eff}(\mathbf{r}) = [2(g_J-1)\mathcal{J}/g\mu_B]\,\mathbf{J}\,\delta(\mathbf{r})$$

$$\mathbf{H}_{eff}(\mathbf{q}) = [2(g_J-1)\mathcal{J}/g\mu_B]\,\mathbf{J}$$

$$\mathbf{s}(\mathbf{r}) = \mathbf{m}(\mathbf{r})/g\mu_B = \frac{1}{g\mu_B V}\sum_{\mathbf{q}}\chi(\mathbf{q})\,\mathbf{H}_{eff}(\mathbf{q})\,e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$= [2(g_J-1)\mathcal{J}/g^2\mu_B^2 V]\sum_{\mathbf{q}}\chi(\mathbf{q})\,e^{i\mathbf{q}\cdot\mathbf{r}}\,\mathbf{J}$$

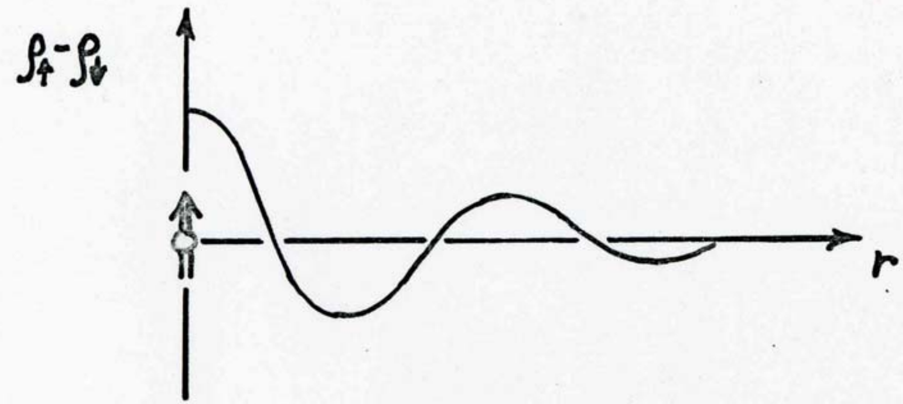
$$\chi(\mathbf{q}) = \chi_P\,F\left(\frac{q}{2k_F}\right)$$

$$= \chi_P\left[\frac{1}{2} + \frac{k_F}{2q}\left(1-\frac{q^2}{4k_F^2}\right)\log\left|\frac{2k_F+q}{2k_F-q}\right|\right]$$

where $\chi_P = 2N(0)\mu_B^2$

$$\mathbf{s}(\mathbf{r}) \sim \left\{\frac{\sin(2k_F r) - 2k_F r\cos(2k_F r)}{(2k_F r)^4}\right\} \sim \frac{\cos(2k_F r)}{(2k_F r)^3}\ ;\ k_F r \gg 1$$

Magnetic ordering via RKKY interaction



Leads to ferromagnetic, antiferromagnetic, or complicated magnetic structures –

$$\mathcal{H}_{RKKY} = -\frac{4(g_J-1)^2 j^2}{g^2 \mu_B^2 V} \sum_{\underset{\sim}{q}} \chi(\underset{\sim}{q}) e^{i \underset{\sim}{q} \cdot \underset{\sim}{r}} \underset{\sim}{J}_i \cdot \underset{\sim}{J}_j$$

↑ ELECTRON BAND STRUCTURE

SUPERCONDUCTIVITY

Magnetically ordered superconductors

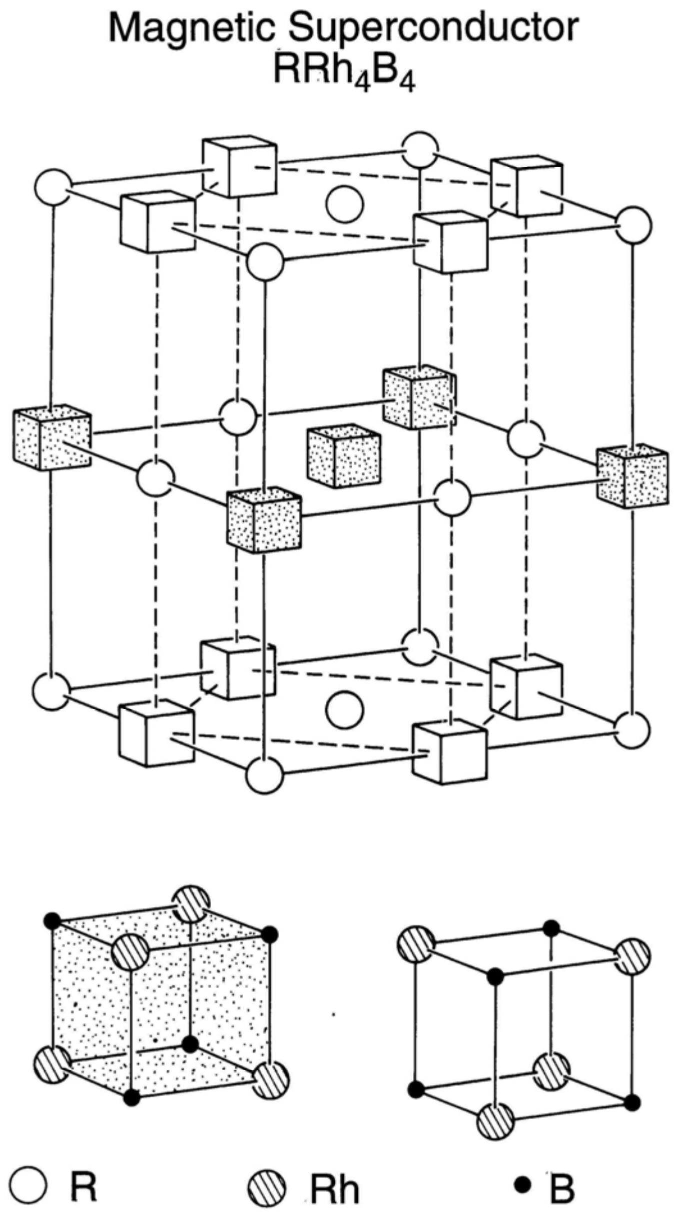
Magnetically ordered superconductors

- Superconducting ternary R Compounds (ordered R sublattice)
  - RMo_6S_8 — *Fischer, Treyvaud, Chevrel, Sergent (75)*
  - RMo_6Se_8 — *Shelton, McCallum, Adrian (76)*
  - RRh_4B_4 — *Matthias, Corenzwit, Vandenberg, Barz (77)*
- Antiferromagnetic superconductors

  Coexistence of SC & AFM
  - RMo_6Se_8 R= Gd,Tb,Er — *UCSD (77)*
  - RMo_6S_8 R= Gd,Tb,Dy,Er — *U. Geneva (77)*
  - RRh_4B_4 R= Nd,Sm,Tm — *UCSD (79)*
  - RNi_2B_2C — *Nagarajan et al. (94); Cava et al. (94)*
  - RNi_2B_2C (single crystals) — *Canfield et al. (94)*
- Ferromagnetic superconductors

  Destruction of SC by onset of FM at $T_{c2} \sim \theta_C < T_{c1}$
  SC-FM interactions – sinusoidally-modulated magnetic state ($\lambda \sim 100$ Å) that coexists with SC near T_{c2}
  - $ErRh_4B_4$ — *UCSD (77)*
  - $HoMo_6S_8$ — *U. Geneva (77)*
  - $ErRh_{1.1}Sn_{3.6}$ — *AT&T, UCSD, BNL (80)*

RRh_4B_4 crystal structure



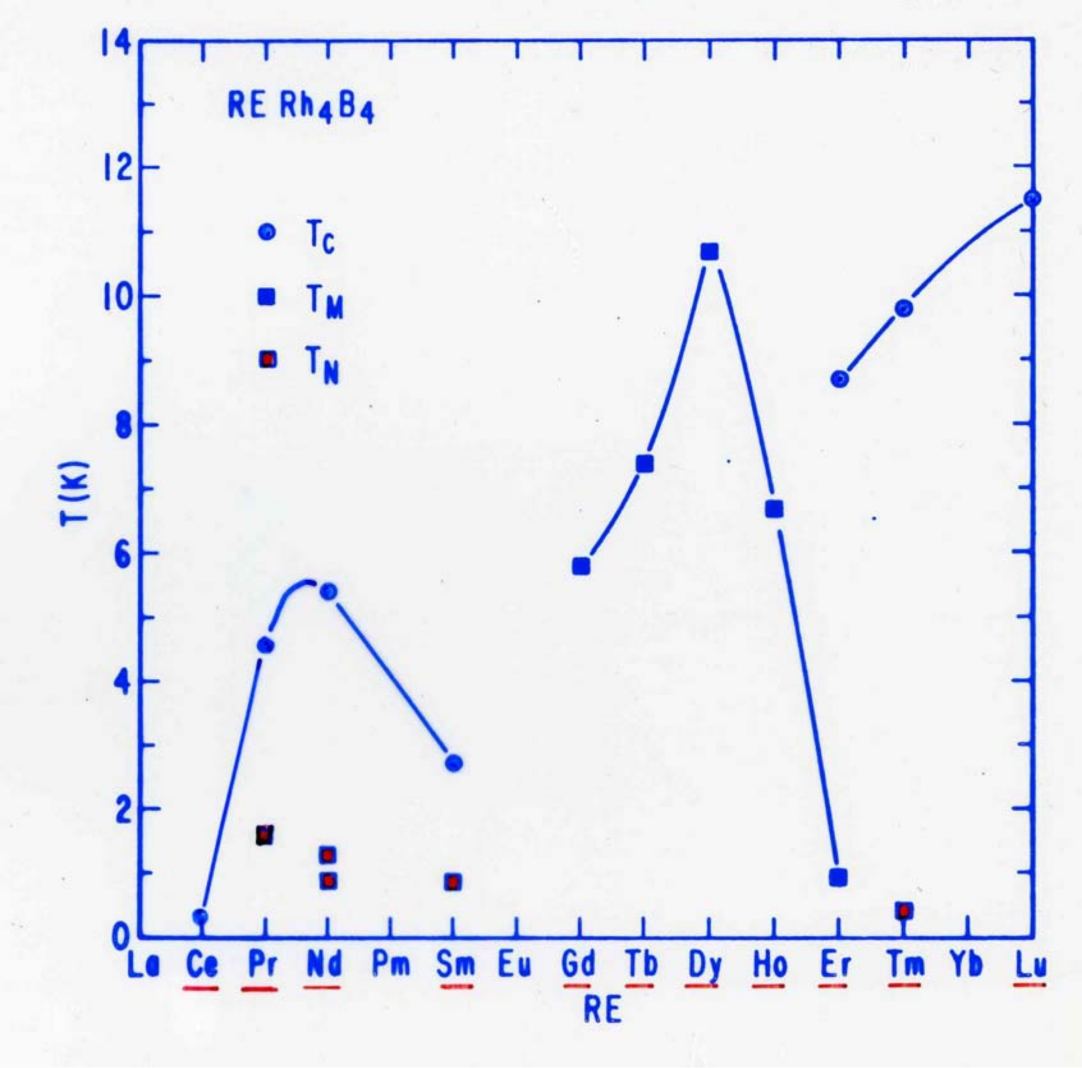
Two weakly interacting subsystems:

RhB “molecular units” or clusters → SC

R magnetic moments → magnetic order

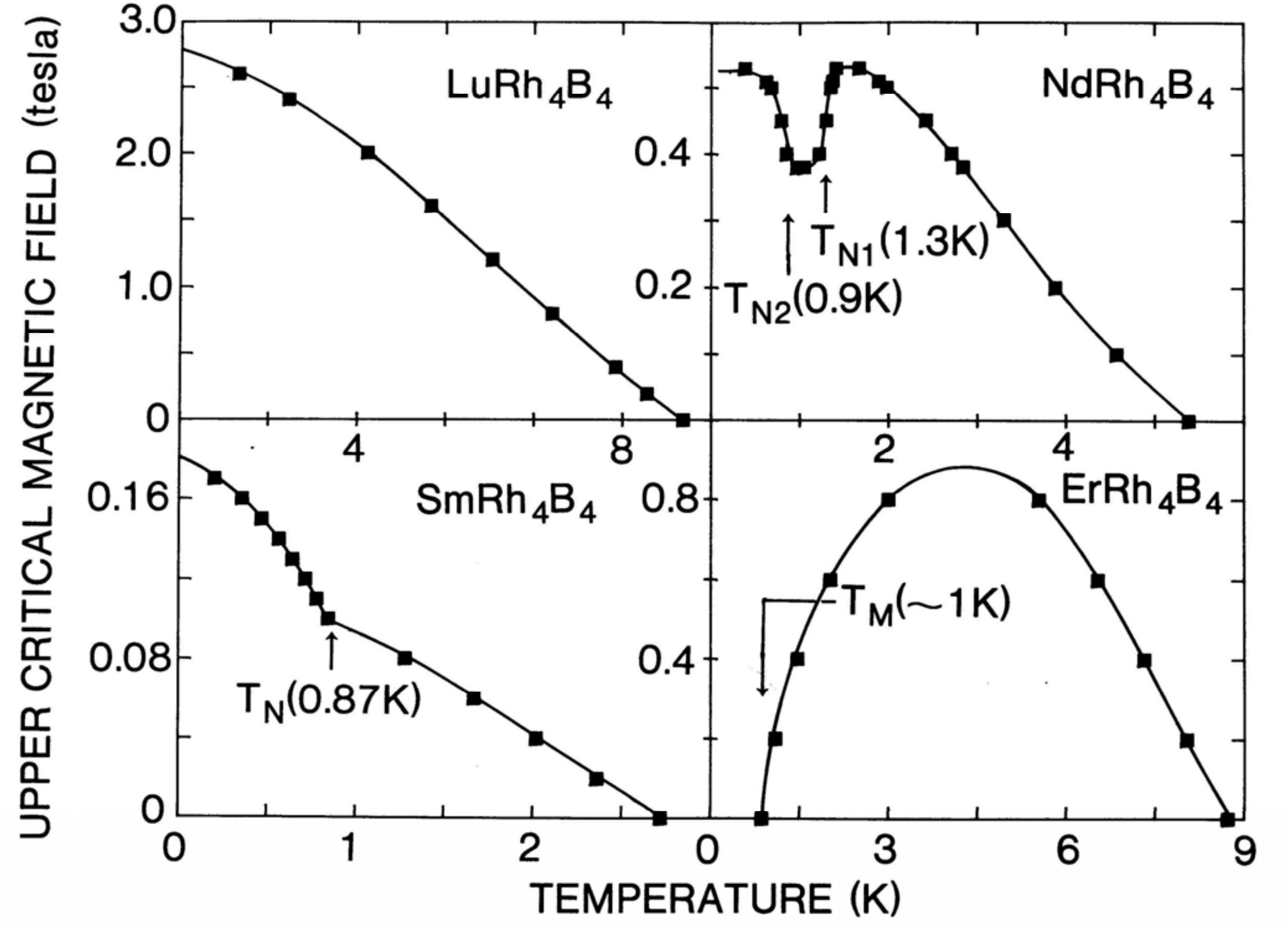
Comparable values of T_c and T_M

Superconducting and magnetic ordering temperatures of RRh_4B_4 compounds



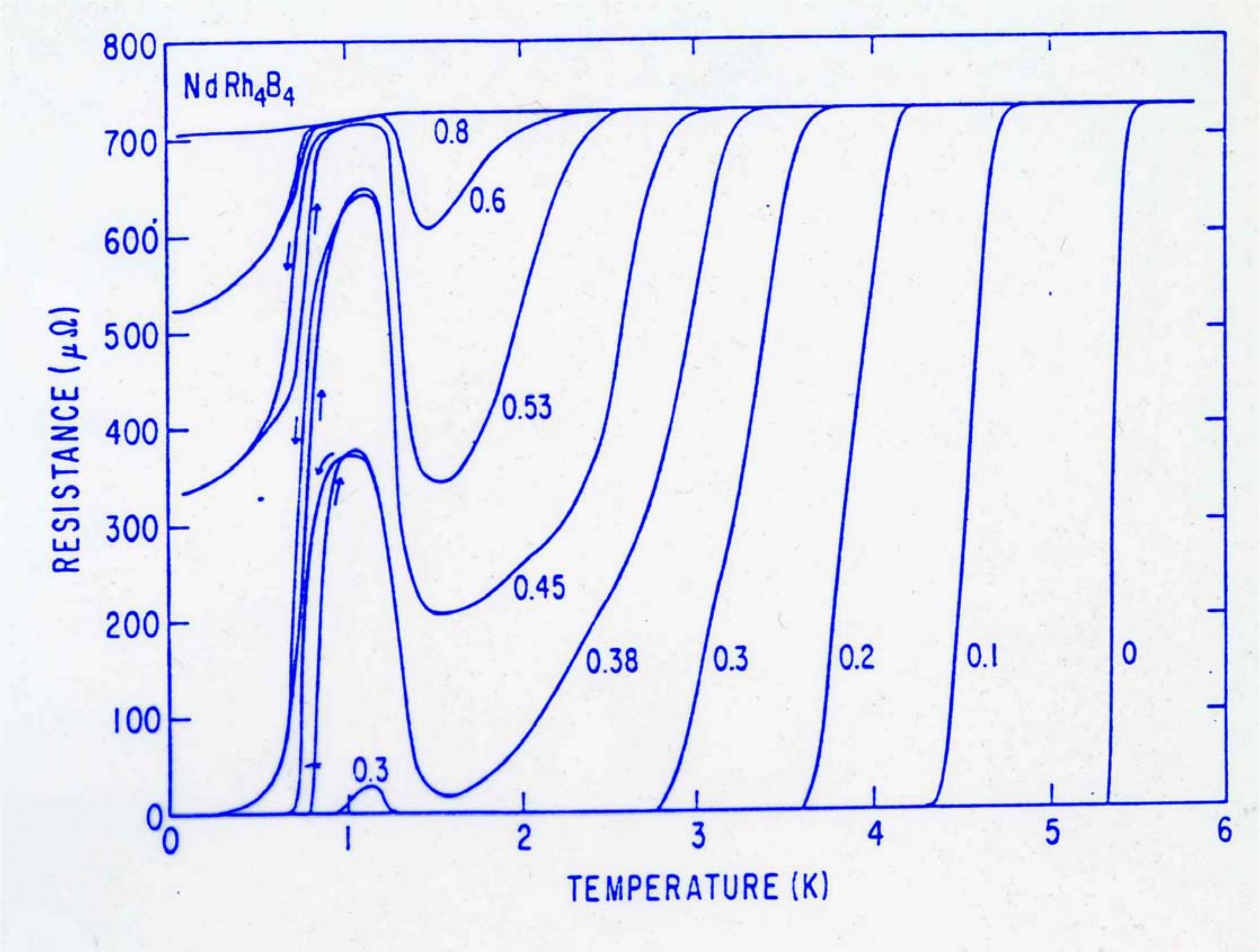
After M. B. Maple, H. C. Hamaker, L. D. Woolf (82)
B. T. Matthias, E. Corenzwit, J. M. Vandenberg, H. Barz (77)
** Ce, Pr – T. Ooyama, K. Kumagai, J. Nakajima, M. Shimotomai (87)*

$H_{c2}(T)$ of RRh_4B_4 magnetic superconductors



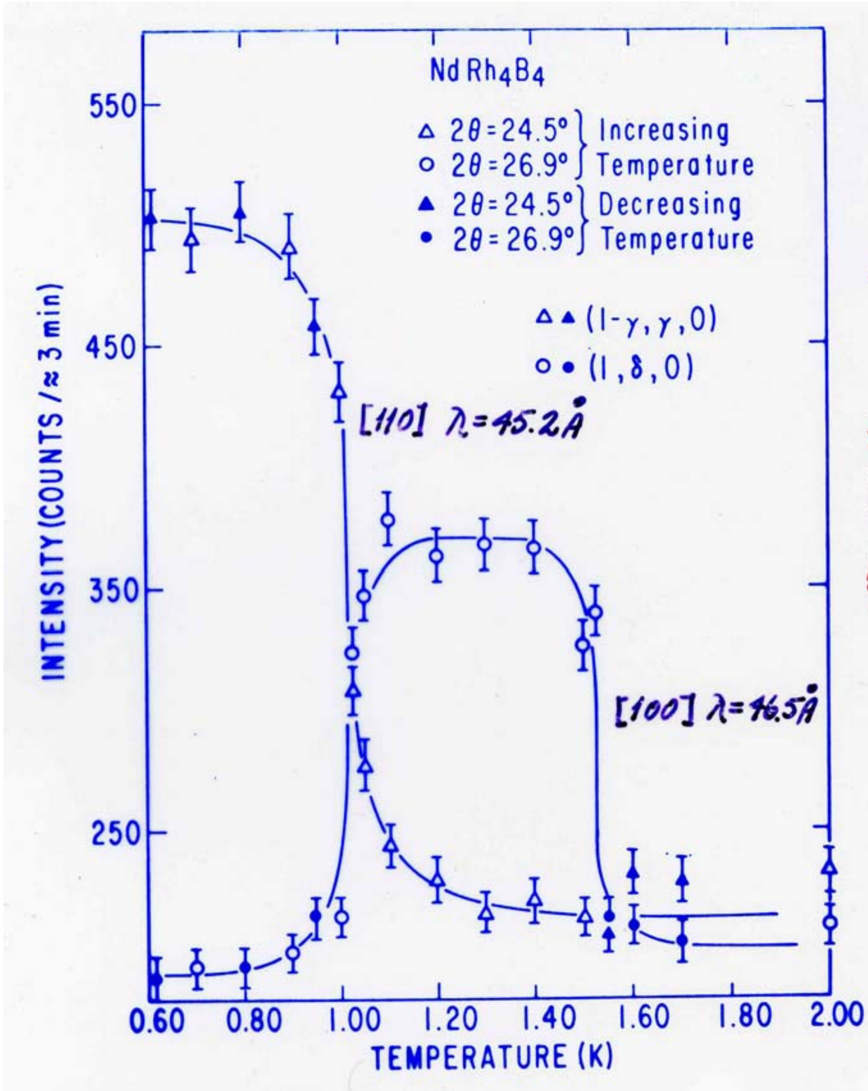
- Nonmagnetic SC: $LuRh_4B_4$
- AFM-SCs: $NdRh_4B_4$, $SmRh_4B_4$
- FM-SC: $ErRh_4B_4$

Magnetic superconductor $NdRh_4B_4$: resistive transition curves



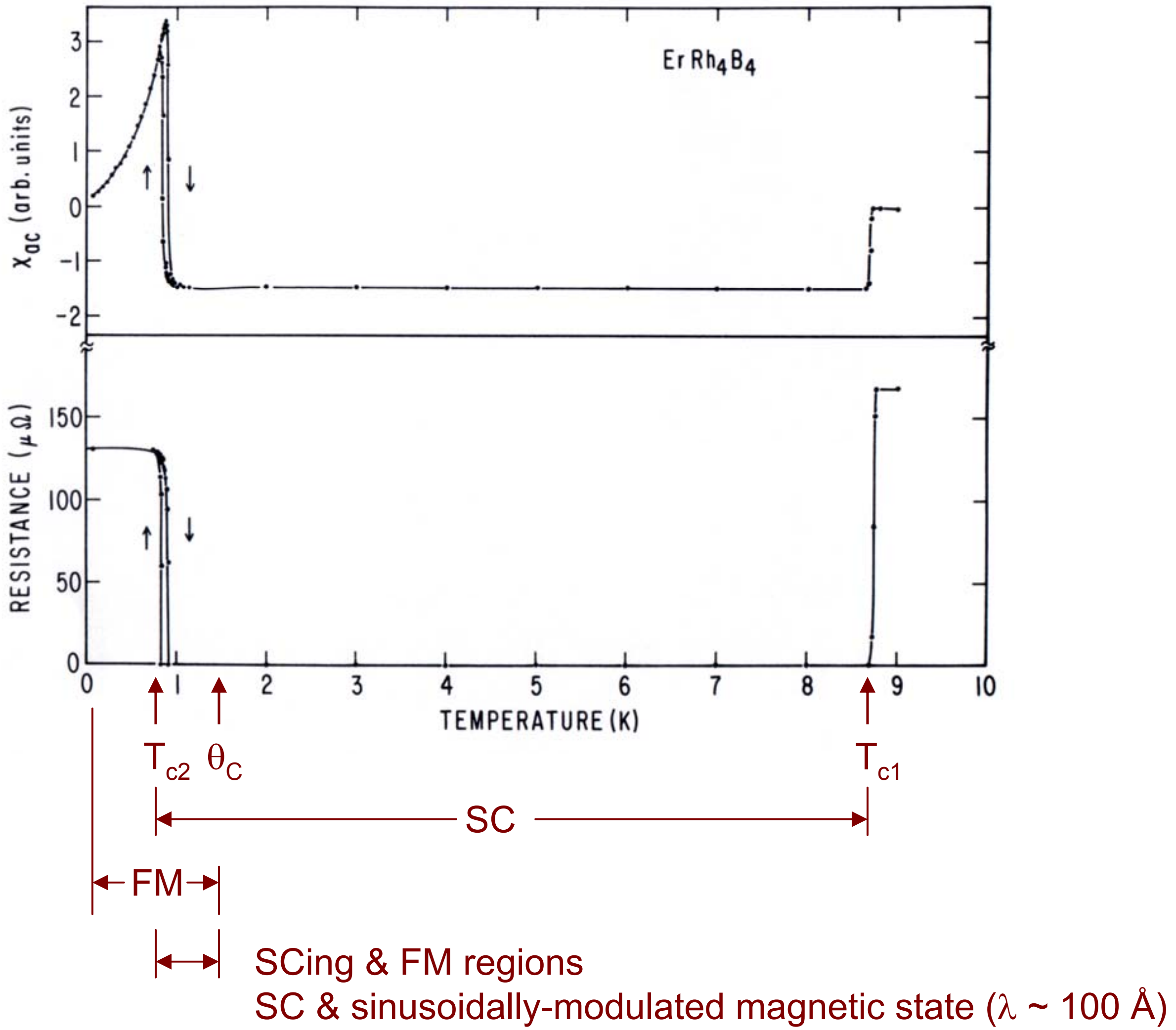
H. C. Hamaker, L. D. Woolf, H. B. MacKay, Z. Fisk, M. B. Maple (79)

Magnetic superconductor $NdRh_4B_4$: neutron scattering



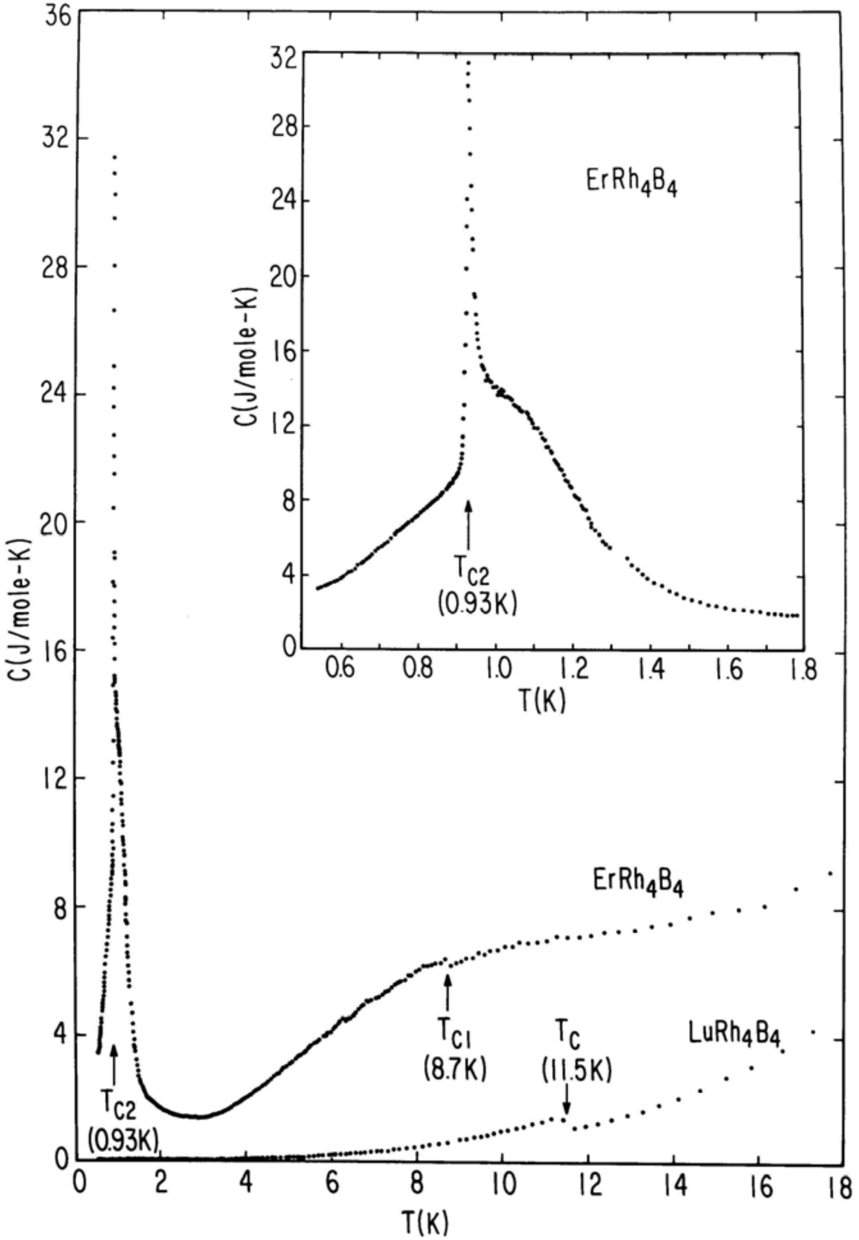
C. F. Majkrzak, D. E. Cox, G. Shirane, H. A. Mook,
H. C. Hamaker, H. B. MacKay, Z. Fisk, M. B. Maple (82)

Reentrant SC due to FM order: $ErRh_4B_4$



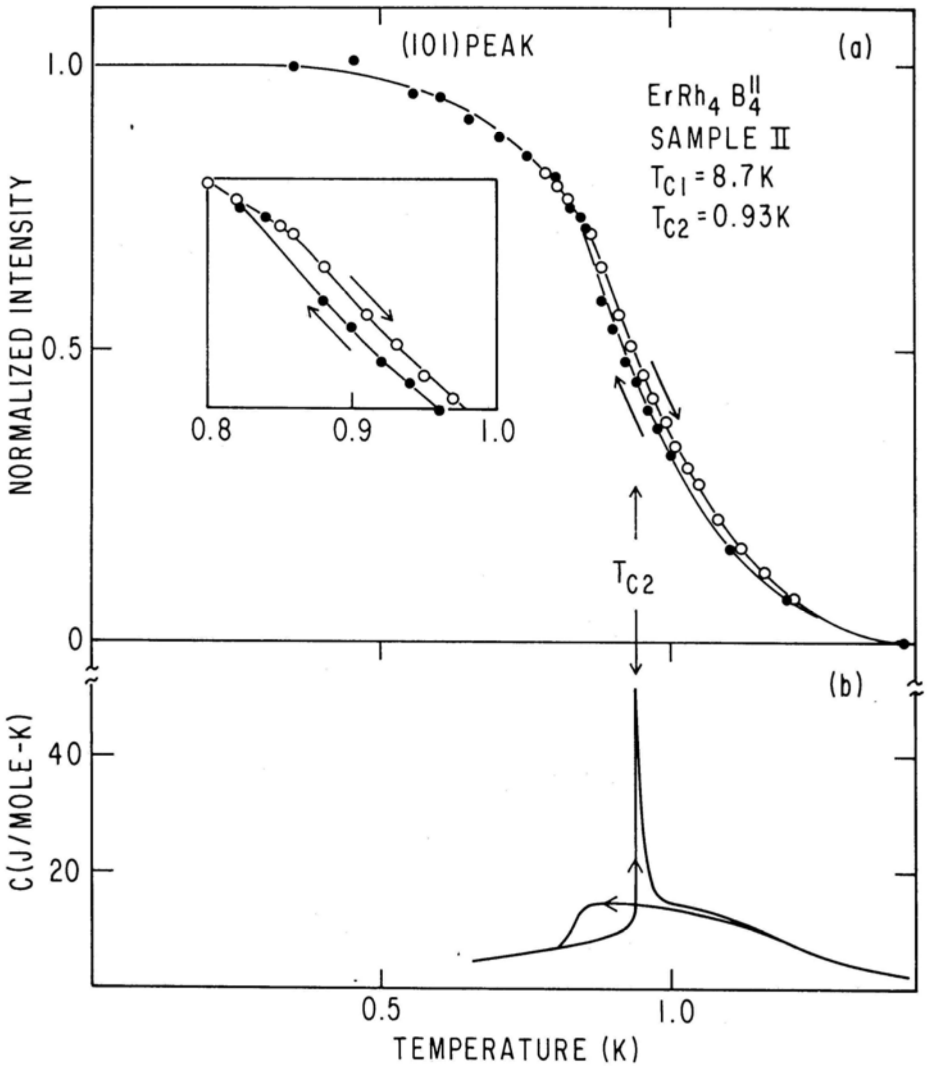
- *Fertig, Johnston, DeLong, McCallum, Maple, Matthias (77)*
- *Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80) (neutron scattering)*

Reentrant FM SC $ErRh_4B_4$: specific heat



L. D. Woolf, D. C. Johnston, H. B. MacKay, R. W. McCallum, M. B. Maple (79)

<u>Macroscopic coexistence</u> of SC & normal FM domains

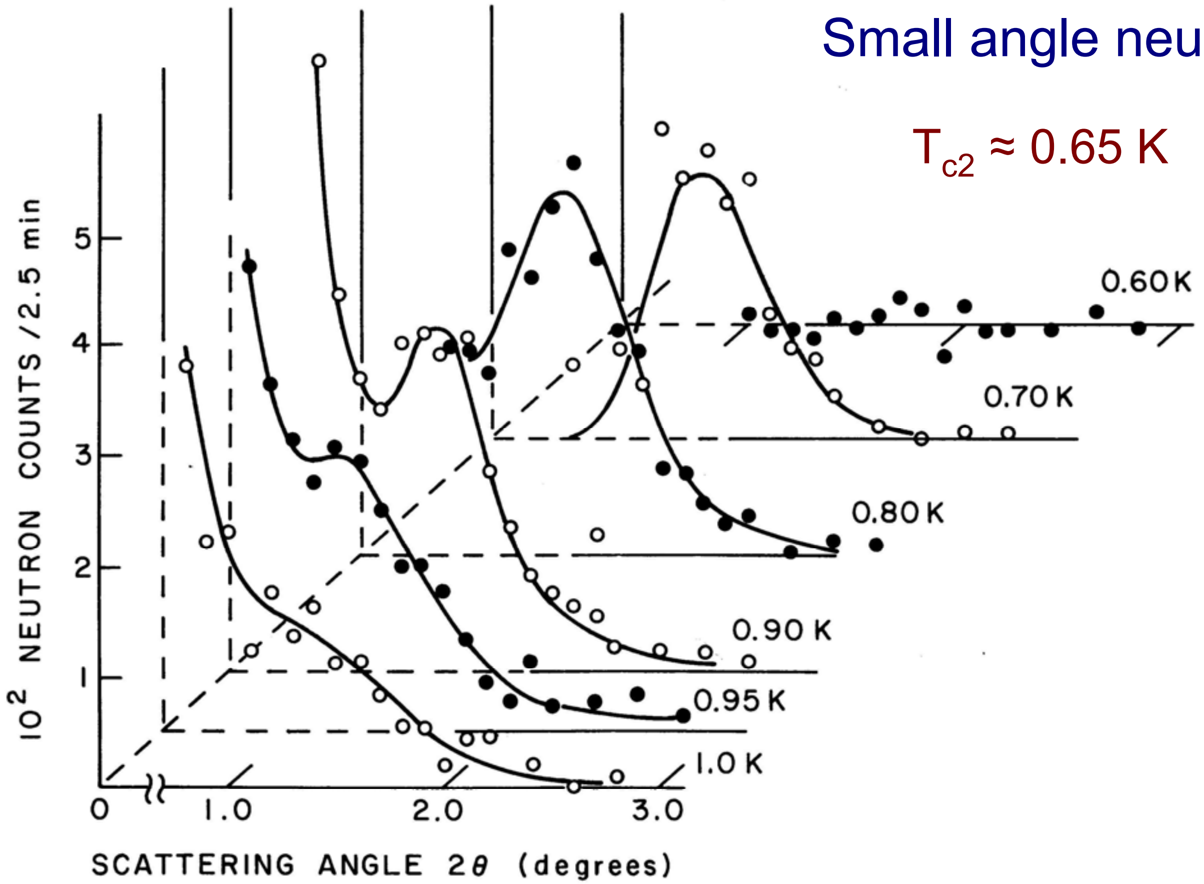


Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80)

<u>Microscopic coexistence</u> of SC & sinusoidally-modulated magnetic state with λ~100 Å

Small angle neutron scattering

$T_{c2} \approx 0.65$ K



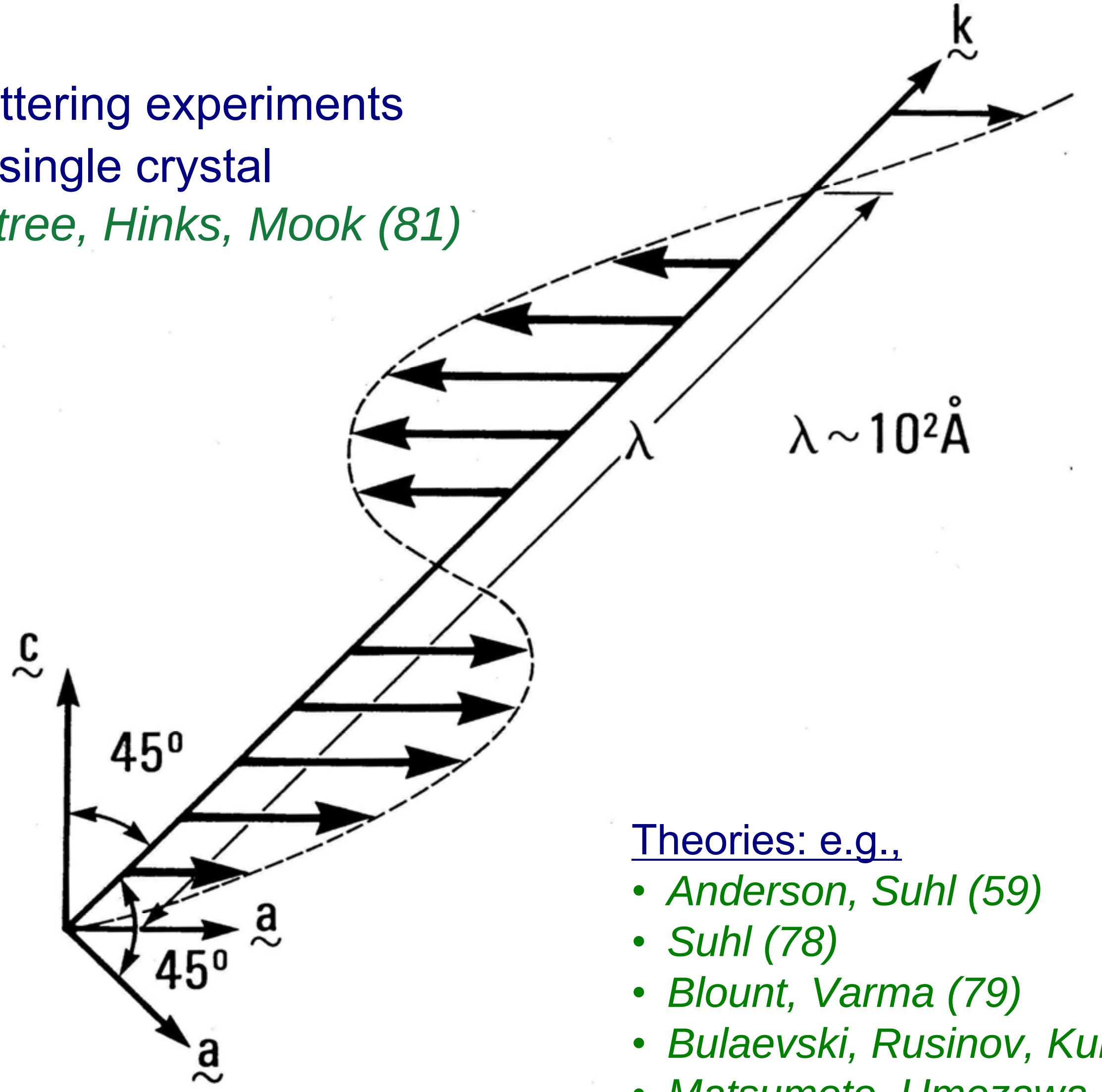
Moncton, McWhan, Schmidt, Shirane, Thomlinson, Maple, MacKay, Woolf, Fisk, Johnston (80)

Similar behavior – $HoMo_6S_8$

Lynn et al. (81)

LINEARLY POLARIZED SINUSOIDALLY MODULATED MAGNETIC STATE ($\mu \perp c$) – $ErRh_4B_4$

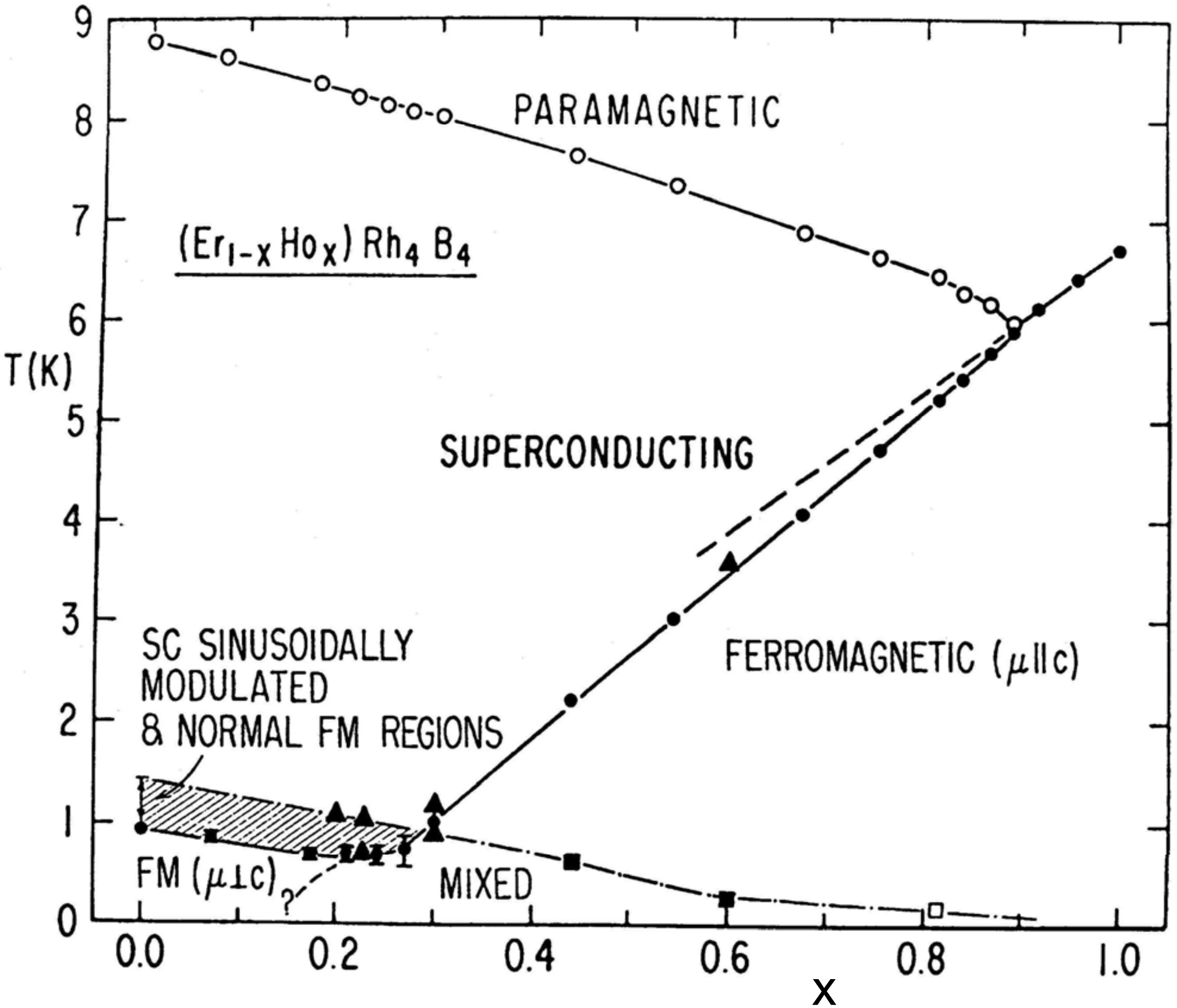
Neutron scattering experiments on $ErRh_4B_4$ single crystal
Sinha, Crabtree, Hinks, Mook (81)



<u>Theories: e.g.,</u>

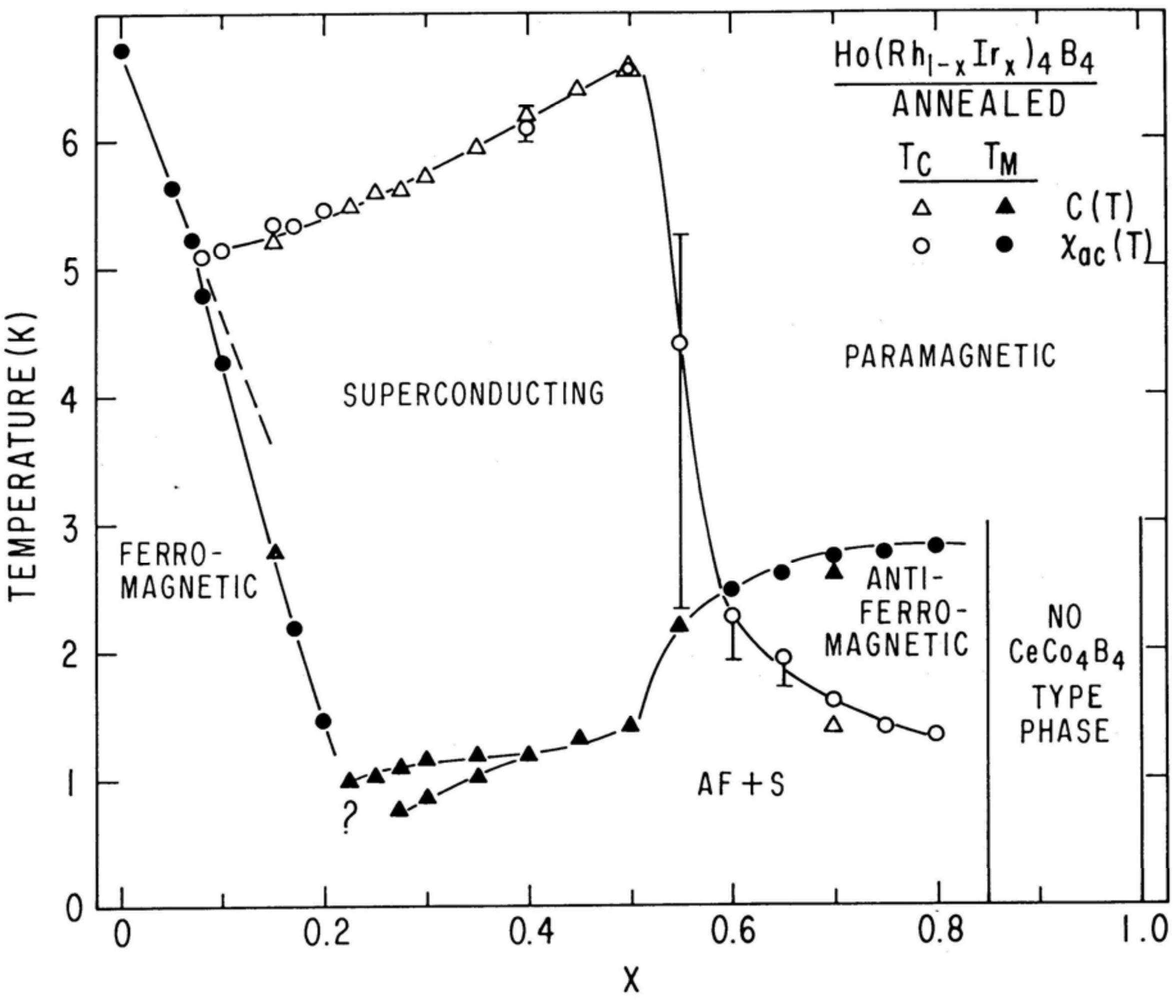
- *Anderson, Suhl (59)*
- *Suhl (78)*
- *Blount, Varma (79)*
- *Bulaevski, Rusinov, Kulik (79)*
- *Matsumoto, Umezawa, Tachiki (79)*

$(Er_{1-x}Ho_x)Rh_4B_4$: FM – $\mu \perp c$ vs $\mu \parallel c$



Johnston, Fertig, Maple, Matthias (78);
Mook, Koehler, Maple, Fisk, Johnston, Woolf (82)

$Ho(Rh_{1-x}Ir_x)B_4$: FM vs AFM



- H. C. Ku, F. Acker, B. T. Matthias (80)
- K. N. Yang, S. E. Lambert, H. C. Hamaker, M. B. Maple, H. A. Mook, H. C. Ku (82)
- S. E. Lambert, M. B. Maple, O. A. Pringle, H. A. Mook (85)

Oscillatory magnetic state in FM SCs

$ErRh_4B_4$, $HoMo_6S_8$: $\lambda \sim 10^2$ □Å (neutron scattering)
Explanation based on electromagnetic interaction
e.g.; Blount & Varma (1979)
Ferrell, Battacharjee & Bagchi (1979)
Matsuda, Umezawa & Tachiki (1979)
Suhl (1980)

$$\mathbf{h}_m(\mathbf{x}) = \gamma(-i\nabla)\,\mathbf{m}(\mathbf{x}) + \mathbf{h}(\mathbf{x})$$

$\mathbf{h}_m(\mathbf{x})$ - molecular field acting on RE ion

$\mathbf{m}(\mathbf{x})$ - RE magnetic moment

$\mathbf{h}(\mathbf{x})$ - magnetic field generated by persistent current

$$\mathbf{h}_m(\mathbf{q}) = \gamma(\mathbf{q})\,\mathbf{m}(\mathbf{q}) + \mathbf{h}(\mathbf{q}) = \gamma(\mathbf{q})\,\mathbf{m}(\mathbf{q}) - 4\pi F(\mathbf{q})\,\mathbf{m}(\mathbf{q})$$

$$= [\gamma(\mathbf{q}) - 4\pi F(\mathbf{q})]\,\mathbf{m}(\mathbf{q}) = \tilde{\gamma}(\mathbf{q})\,\mathbf{m}(\mathbf{q})$$

Oscillatory magnetic state in FM SCs

Normal:

$\chi(\underset{\sim}{q}) = (T_M - Dq^2)/C$

T_M - Curie temperature

D - magnetic stiffness coefficient

$C = N\mu_{eff}^2/3k_B$ - Curie constant

$\chi(\underset{\sim}{q})$ maximum at $q = 0 \Rightarrow \lambda = \infty$

$\Rightarrow$ FM for $T \leq T_M$ (2nd order)

Superconducting:

$\tilde{\chi}(\underset{\sim}{q}) = \chi(\underset{\sim}{q}) - 4\pi F(\underset{\sim}{q})$; $F(\underset{\sim}{q}) = \dfrac{\exp(-\xi^2 q^2/2)}{\lambda_L^2 q^2 + \exp(-\xi^2 q^2/2)}$

ξ - coherence length

λ_L - London penetration depth

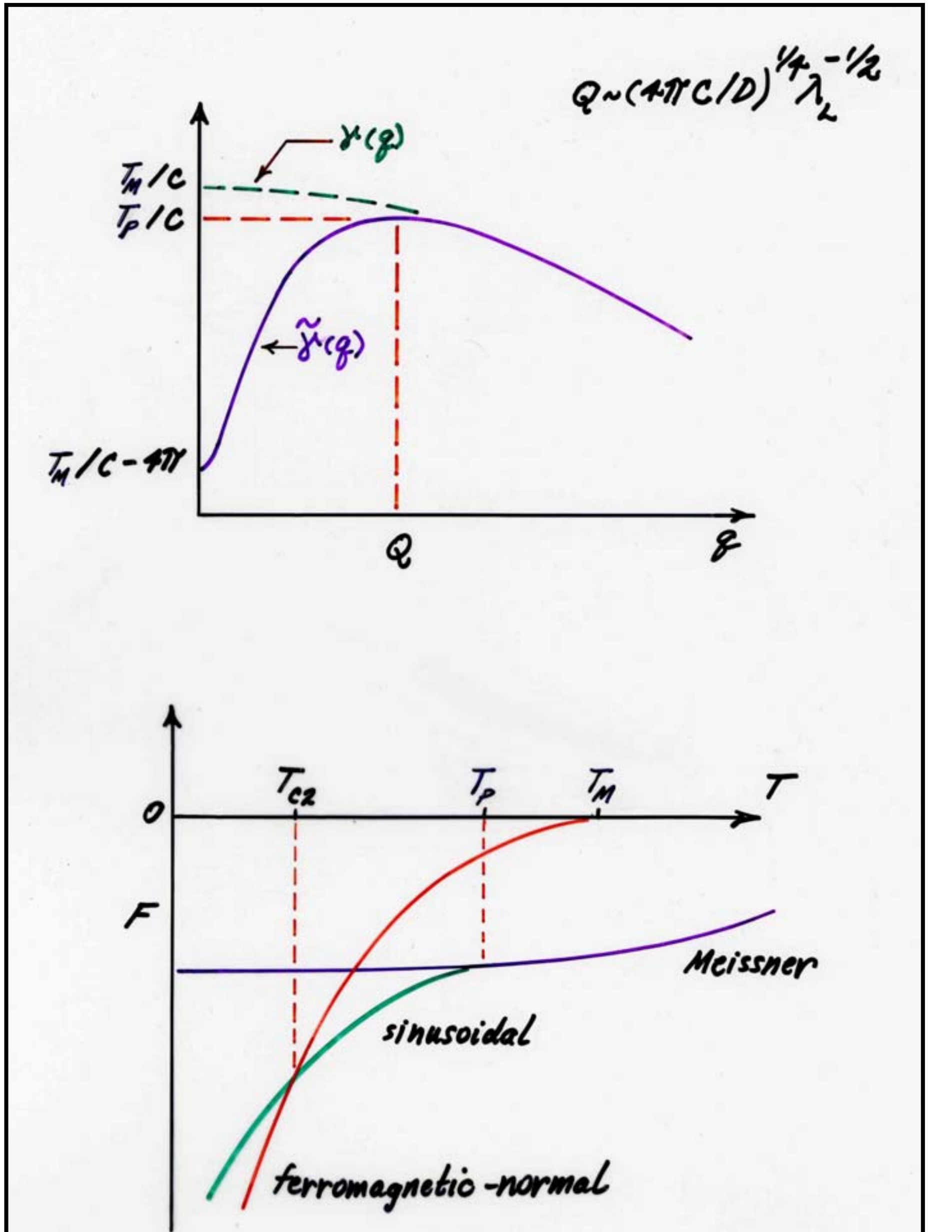
$\tilde{\chi}(\underset{\sim}{q})$ maximum at $Q \sim (4\pi C/D)^{1/4} \lambda_L^{-1/2}$

SC screens exchange interaction at long wavelengths

$\Rightarrow$ AFM & SC for $T_{C2} \leq T \leq T_P < T_M$ (2nd order)

FM for $T \leq T_{C2}$ (1st order)

Oscillatory magnetic state in FM SCs



END

Superconductivity

- Conventional superconductivity

  Electron pairs (Cooper pairs) – ($\mathbf{k}\uparrow$,$-\mathbf{k}\downarrow$)

  S = 0 (singlet)

  L = 0 (s-wave)

  Pairing mechanism — electron-phonon interaction

  $T_c \approx \theta_D \exp(-1/N(E_F)V)$

  Isotropic energy gap $\Delta(\mathbf{k}) = \Delta$

  "Activated" behavior;

  e.g., $C_e(T) \sim \exp(-\Delta/T)$

Superconductivity

- <u>Unconventional superconductivity</u>

  Electron pairs (L > 0)

  S = 1 (triplet) $\Rightarrow$ L = 1 (p-wave)

  S = 0 (singlet) $\Rightarrow$ L = 2 (d-wave)

  Pairing mechanism – AFM spin fluctuations

  Anisotropic energy gap $\Delta(\mathbf{k}) \neq \Delta$

  Δ(k) vanishes at points or lines on Fermi surface

  "Power Law" behavior;

  e.g., $C_e(T) \sim T^n$ (n = 2, line nodes; n = 3, point nodes)