

SC discovered in SRO by Maeno in 1994

Chiral p-wave superconductivity and Sr₂RuO₄

Catherine Kallin, McMaster

ICMR Summer School on Novel Superconductors Santa Barbara, August 12, 2098

Chiral p-wave & SRO Outline

- Introduction to SRO and early evidence for chiral p-wave superconductivity
- Chiral p-wave SC order
 - triplet pairing
 - topological order & Majorana fermions
 - spontaneous supercurrents
- Search for spontaneous supercurrents & attempts to reconcile with theory
- Polar Kerr effect experiments & theory
- Phase sensitive measurements
- Putting it all together

Strontium Ruthenate





- Same structure as La2CuO4 cuprate
- Quasi-two-dimensional; multiband
- T_c ≤1.5K, strongly disorder dependent
 - \rightarrow unconventional pairing

Singlet or Triplet?

Spin susceptibility measurement can distinguish between singlet and triplet superconductivity. For spin singlet pairing:

$$\chi_s / \chi_N = -\frac{2}{N} \int_0^\infty dE \frac{\partial f(E)}{\partial E} N_s(E) \rightarrow e^{-\Delta/k_B T}$$
 at low T for s - wave



NMR evidence for triplet pairing in SRO



Knight shift → triplet pairing
[B || ab]

The Knight shift for oxygen-17 in YBCO (left) and SRO (right). For spin-singlet Cooper pairing, Knight-shift data exhibit a drop in the spin susceptibility in the superconducting state. Such a drop occurs in YBa2Cu3O7, but not in Sr2RuO4, whose superconductivity is most likely mediated by spin-triplet Cooper pairs. From K. Ishida et al., *Nature* **396**, 658 (1998).





Actual data from Ishida et al. Nature (1998).

Triplet→ p-wave pairing most likely/simplest possibility

More recent NMR data less clear.





Figure 1 Zero-field μ SR spectra measured with $P_{\mu} \pm c$ in Sr₂RuO₄ at T = 2.1 K (circles) and T = 0.02 K (squares). Curves are fits to the product of equation (1) and exp(-At) as described in the text.

G. M. Luke*, Y. Fudamoto*, K. M. Kojima*, M. I. Larkin*, J. Merrin*, B. Nachumi*, Y. J. Uemura*, Y. Maeno†, Z. Q. Mao†, Y. Mori†, H. Nakamura‡ & M. Sigrist§ Nature 394, 558 (1998).



Figure 2 Zero-field (ZF) relaxation rate *A* for the initial muon spin polarization lc (top) and $\pm c$ (bottom). T_c from a.c.-susceptibility indicated by arrows. Circles in bottom figure give relaxation rate in $B_{LF} = 50 \text{ G} \pm c$. Curves are guides to the eye.

muSR sees internal fields (as one would see in a ferromagnet or antiferromagnet – but dilute in this case) which turn on with superconducting order. This is a signature of broken time reversal symmetry

$$\mathcal{T}\Psi_{\sigma} \rightarrow \Psi_{\sigma}^{*}$$

Can break TRS with spin order or with orbital order.

Early expts pointed toward triplet (simplest case is p-wave) with TRSB.

Singlet SC

$$S = 0 \Rightarrow \chi_{s=0} = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)$$
order parameter: $\psi = \Delta(\vec{k})\chi_{s=0}$
s - wave: $\Delta(\vec{k}) = \Delta_0$ (simplest case; may have k - dependence)
d - wave: $\Delta(\vec{k}) = \begin{cases} [k_x^2 - k_y^2]/k_F^2 \\ [(k_x^2 - k_y^2) + ik_xk_y]/k_F^2 \end{cases}$ (for example)

 $S = 1 \Rightarrow |\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle \Rightarrow \text{must specify pairing amplitude}$ for each triplet state

Triplet SC
$$S = 1 \Rightarrow |\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), |\downarrow\downarrow\rangle$$

order parameter:
$$\psi = \Delta_{\uparrow\uparrow}(\vec{k})\chi_{\uparrow\uparrow} + \Delta_{\downarrow\downarrow}(\vec{k})\chi_{\downarrow\downarrow} + \Delta_{\uparrow\downarrow}(\vec{k})(\chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow})$$

can organize this into a 2x2 matrix form: $\psi = (\uparrow,\downarrow)_1 \begin{pmatrix} \Delta_{\uparrow\uparrow} & \Delta_{\uparrow\downarrow} \\ \Delta_{\uparrow\downarrow} & \Delta_{\downarrow\downarrow} \end{pmatrix} \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}_2$

or can organize into 3-component vector (Balian - Werthamer, 1963):

$$\psi = i\{\vec{d}(\vec{k}) \cdot \vec{\sigma}\}\sigma_y \quad \text{where } \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
$$\psi = i\{d_x\sigma_x + d_y\sigma_y + d_z\sigma_z\}\sigma_y = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

Triplet SC continued

$$\psi = i\{\vec{d}(\vec{k}) \cdot \vec{\sigma}\}\sigma_y = \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

If
$$\vec{d} = d_z \hat{z} \Rightarrow \psi = d_z (|\uparrow \downarrow \rangle + |\downarrow \uparrow \rangle) \Rightarrow S_z = 0$$

If \vec{d} is a real vector except for a k - dependent phase, i.e. $\vec{d} \times \vec{d}^* = 0$ (unitary) then the component of *S* along \vec{d} is zero.

What d-vector is compatible with experiments on SRO?

What p-wave states are allowed for SRO by symmetry?

Unitary states ³He d/Δ_0 Δ/Δ_0 Node Time-reversal symmetry $\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y$ $\sqrt{k_{x}^{2}+k_{y}^{2}}$ BW $\hat{\mathbf{x}}k_{y} - \hat{\mathbf{y}}k_{x}$ $\hat{\mathbf{x}}k_x - \hat{\mathbf{y}}k_y$ unitary & gapped & TRSB $\hat{\mathbf{x}}k_{y} + \hat{\mathbf{y}}k_{x}$ $\sqrt{k_{x}^{2}+k_{y}^{2}}$ $\hat{z}k_x$ k_x line $\hat{z}(k_x+k_y)$ $|k_x+k_y|$ line ABM $\hat{z}(k_x \pm ik_y)$ $\sqrt{k_{x}^{2}+k_{y}^{2}}$ broken Nonunitary states $\hat{\mathbf{x}}k_x + i\hat{\mathbf{y}}k_y$ $|k_x+k_y|\uparrow\uparrow$ broken $|k_x - k_y| \downarrow \downarrow$ $\hat{\mathbf{x}}k_y - i\hat{\mathbf{y}}k_x$ $|k_y - k_x|\uparrow\uparrow$ broken (unitary: $dxd^{*}=0$) $|k_x + k_y| \downarrow \downarrow$ $\hat{\mathbf{x}}k_x - i\hat{\mathbf{y}}k_y$ $|k_y - k_x|\uparrow\uparrow$ broken $|k_x+k_y|\downarrow\downarrow$ $\hat{\mathbf{x}}k_{y} + i\hat{\mathbf{y}}k_{x}$ $|k_x+k_y|\uparrow\uparrow$ broken $|k_{y}-k_{x}|\downarrow\downarrow$ $(\hat{\mathbf{x}}+i\hat{\mathbf{y}})(k_x+k_y)$ $2(|k_x+k_y|)\uparrow\uparrow$ line broken 0 [] $(\hat{\mathbf{x}}+i\hat{\mathbf{y}})(k_x+ik_y)$ $2\sqrt{k_x^2+k_y^2}\uparrow\uparrow$ broken A10 []

TABLE IV. Allowed *p*-wave states on a cylindrical Fermi surface for a tetragonal crystal. References to ³He are for the analogous three-dimensional states.

[from Mackenzie & Maeno, RMP (2003).]

Superfluid ³He

Lee, Osheroff & Richardson (1971)



Chiral p-wave Superconductivity

$$\mathbf{d}(\mathbf{k}) = \Delta_{0} \hat{\mathbf{z}}(k_{x} \pm ik_{y})$$

Breaks time-reversal symmetry

 k_x +i k_y degenerate with k_x -i k_y \rightarrow can have domains

$$\vec{L} = \hbar \hat{\mathbf{z}}$$



Both d-vector and L aligned along c-axis equal spin pairing in ab-plane

Each Cooper pair carries angular momentum ħ and the BCS SC state carries $\langle L_z \rangle = \hbar N/2$ (Stone & Roy, PRB 2004.)

Zero modes & Majorana Fermions

The chiral p-wave state is equivalent to the Moore-Read quantum Hall 5/2 state \rightarrow topological state with special Majorana fermion edge modes (also in vortex cores). See Read & Green, PRB (2000); Gurarie & Radzihovsky, Ann. Phys. (2007).

Consider mean field Hamiltonian for superconductivity:

$$H_{MF} = \sum_{ij} \left\{ h_{ij} a_i^{\dagger} a_j + h_{ji} a_j^{\dagger} a_i \right\} + \Delta_{ij} a_i^{\dagger} a_j^{\dagger} + \Delta_{ij}^{\dagger} a_i a_j$$
$$= \sum_{ij} \left(a_i^{\dagger} a_i \right) \underbrace{\begin{pmatrix} h_{ij} & \Delta_{ij} \\ \Delta_{ij}^{\dagger} & -h_{ij}^{T} \end{pmatrix}}_{\mathrm{H}} \begin{pmatrix} a_j \\ a_j^{\dagger} \end{pmatrix} \quad \text{where } \Delta_{ji} = -\Delta_{ij}$$

The Bogoliubov-de Gennes equations are: $H\psi_n = E_n\psi_n$

$$H\psi_n = E_n \psi_n$$
 where $H = \begin{pmatrix} h_{ij} & \Delta_{ij} \\ \Delta_{ij}^+ & -h_{ij}^T \end{pmatrix}$. Let $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Note:
$$\sigma_1 H \sigma_1 = -H^* \Rightarrow \text{ if } H \psi_n = E_n \psi_n$$

then $H \sigma_1 \psi_n^* = \sigma_1^2 H \sigma_1 \psi_n^* = -\sigma_1 H^* \psi_n^* = -E_n \sigma_1 \psi_n^*$
 $\psi_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix} \Rightarrow \sigma_1 \psi_n^* = \begin{pmatrix} v_n^* \\ u_n^* \end{pmatrix}$

Bogoliubov qps:
$$\gamma_n^+ = \sum_i \left(u_{ni} a_i^+ + v_{ni} a_i \right)$$
 and $H = \sum_n E_n \gamma_n^+ \gamma_n$.
If $H\psi = 0$, then $H\sigma_1 \psi^* = 0$. Let $\psi_0 = \psi + \sigma_1 \psi^* \Rightarrow H\psi_0 = 0$
Then $\sigma_1 \psi_0^* = \psi_0 \Rightarrow \psi_0 = \begin{pmatrix} u_0 \\ u_0^* \end{pmatrix} \Rightarrow \gamma^+ = \sum_i u_{0i} a_i^+ + u_{0i}^* a_i = \gamma$.

 $\gamma = \gamma^+ \Rightarrow$ its own antiparticle need 2 to make 1 fermion $c = \gamma_{01} + i\gamma_{02}$ $c^+ = \gamma_{01} - i\gamma_{02}$ {c,c⁺} = 1

These zero modes are **Majorana fermions** and must come in pairs.

chiral p-wave: a single Majorana fermion can exist at a vortex core or an edge and, if well separated, are protected by the bulk gap. Need to analyze spatially varying BdG eqs for chiral p-wave. See, for example, Gurarie & Radzihovsky, Ann. Phys. (2007). Need conditions for half-quantum vortices to isolate single Majorana fermions.



From Stone & Anduaga, PRB (2007). Energy levels as function of angular momentum for chiral p-wave in harmonic trap.

Exotic physics from chiral p-wave



d-vector perpendicular to z

→ may have half-quantum vortices with a single
 Majorana zero mode bound at the core

 \rightarrow non-Abelian statistics.

→ useful for quantum computing because of the topological stability and nontrivial winding

NMR for H z may be explained by rotating d-vector



[From Ishida (KITP 2007)]

More recent NMR with H || c also found no supression of Knight shift below Tc

 \rightarrow triplet with spins along c ?

→ not necessarily same orbital state

→ NMR lineshapes broader



Spontaneous supercurrents

$$\mathbf{d}(\mathbf{k}) = \Delta_0 \hat{\mathbf{z}}(k_x \pm ik_y)$$

p-wave \rightarrow defects and surfaces are pair-breaking

Chiral p-wave → disturbances cause supercurrents



Stone and Roy (2004) Matsumato and Sigrist (1999)

$$\left\langle L_{z}\right\rangle =\hbar N/2$$



Mid-gap bound states plus scattering states give a large supercurrent within ξ of surface, which is screened within λ of surface



Any surface is pair-breaking for some component of p_x+ip_v

 $\vec{j} \propto \operatorname{Im}(\psi^* \vec{\nabla} \psi) \Rightarrow$ Supercurrents flow at defects \rightarrow local magnetic fields **Spontaneous supercurrents**

$$\mathbf{d}(\mathbf{k}) = \Delta_0 \mathbf{\hat{z}}(k_x \pm ik_y)$$

p-wave \rightarrow defects and surfaces are pair-breaking

Order parameter is complex (carries angular momentum) → disturbances cause supercurrents



Stone and Roy (2004) Matsumato and Sigrist (1999)

$$\left\langle L_{z}\right\rangle =\hbar N/2$$



Mid-gap bound states plus scattering states give a large supercurrent within ξ of surface, which is screened within λ of surface

Supercurrents in
$$p_x + ip_y$$
SC from GL theoryGinzburg-Landau Free energy: $\mathbf{d}(\mathbf{k}) = \hat{z} |\Delta_0|(u,v)$

$$\begin{split} F &= \frac{1}{4\pi} H_c^2 \tilde{\xi}^3 \int d^3 r \left[-\frac{1}{2} (|u|^2 + |v|^2) + (\frac{1}{8} + \frac{1}{2} \tilde{\beta}_2) (|u|^2 + |v|^2)^2 + \frac{1}{2} \tilde{\beta}_2 (u^* v - uv^*)^2 \right. \\ &\quad \left. - \frac{\tilde{\beta}_3}{8} (|u|^2 - |v|^2)^2 + k_1 (|d_x u|^2 + |d_y v|^2) + k_2 (|d_y u|^2 + d_x v|^2) \right. \\ &\quad \left. + \tilde{k} [(d_x u)^* (d_y v) + (d_x v)^* (d_y u) + \text{c.c.}] + \Delta \tilde{k} [(d_x u)^* (d_y v) - (d_x v)^* (d_y u) + \text{c.c.}] \right. \\ &\quad \left. + k_5 (|d_x u|^2 + |d_x v|^2) + \kappa^2 \mathbf{b}^2 \right] \,, \end{split}$$

.

This implies that a surface at x=0 will cause a supercurrent to flow along y.



$$\begin{aligned} j_y &= \frac{1}{\kappa^2} \operatorname{Im}[k_2 u^* \partial_y u + k_1 v^* \partial_y v + \widetilde{k} (u^* \partial_x v + v^* \partial_x u) \\ &- \Delta \widetilde{k} (u^* \partial_x v - v^* \partial_x u)] \end{aligned}$$



Matsumoto and Sigrist, J. Phys. Soc. Jap. 68, 994 (1999).

Domain wall similar to two edges put together, with opposite sign field on each side and a current flowing along domain wall with compensating currents on each side. In this case, the field maximum is ~ 20 G (whereas the maximum field at an edge is ~ 10 G).

ZF-muSR Decay Rate vs Temperature

- Spontaneous field seen below T_c, for P_m//c, //a
- $B_{loc} \sim 1G$
- Follows T_c
- Fit with Lorentzian; no evidence of same field at each muon



Scanning SQUID microscopy: search for edge and domain currents



Fig. 5. Electric currents (solid lines) of the chiral edge states along the inner and outer edges of Sr₂RuO₄. The dashed lines show superfluid counterflow. The rectangular coil on the right represents a SQUID pickup loop.

Scanning SQUID microscopy: search for edge and domain currents

J.R. Kirtley, C. Kallin, C. Hicks, E.A. Kim, Y. Liu, K.A. Moler, Y. Maeno, PRB (2007).











FIG. 1. A SEM image of the Sr_2RuO_4 crystal used for the Hall probe imaging. An array of holes was milled in the sample using a FIB. The hole spacing is 20 μ m.

Scanning Hall probe measurements (K.A. Moler's group)



FIG. 2. (Color online) Scanning Hall probe image at T=100 mK of the sample shown in Fig. 1, cooled in a background field of ~25 mG. The background has been line normalized to remove 1/f noise. (a) Image with a color scale showing the full measured magnetic field range. Isolated trapped vortices dominate the image. (b) Cross section taken along the darkened line in (a). (c) Image with an expanded color scale, showing that there are no obvious features in the noise. (d) Cross section taken along the lightened line in (c).

Predicted signals from Matsumoto & Sigrist plus modelling for expt. setup



Effects on supercurrents

- Multibands
- Anisotropy (mixing in higher harmonics to p-wave)
- Disorder
- Leggett's wavefunction. <L_z> reduced

All of the above will reduce all supercurrents (domain walls, impurities) and so would reduce expected muSR signature as well

 Need to look for surface effects such as roughness, pairbreaking, nucleating other order parameters. Considerations not the same for edge currents and for observing domain walls intersecting ab surface. EFFECTS OF SURFACE ROUGHNESS, PAIRBREAKING, GL PARAMETERS



Polar Kerr effect



Linearly polarized light is reflected as elliptically polarized light, with rotation of polarization axis by Kerr angle

Measure magnetization perpendicular to surface in FM

Requires broken time-reversal symmetry



Cooled in (a) 93 G (b) -43 G [ω =0.8ev; Θ =60 nanorads]

J. Xia, Y. Maeno, P.T. Beyersdorf, M.M. Fejer, A. Kapitulnik, PRL 97, 167002 (2006).

Polar Kerr effect

$$\theta_{K} = \frac{4\pi\sigma_{xy}''(\omega)}{n(n^{2}-1)\omega} \propto \sigma_{xy}''(\omega) \qquad \text{(for } \omega > \omega_{p}\text{)}$$

Absorptive part of $\sigma_{xy} \rightarrow$ system absorbs either right or left circularly polarized light preferentially

However, in the absence of disorder

$$\vec{j}_s = \frac{ie^2 \rho_s / m}{\omega + i\delta} \vec{E} \Longrightarrow \sigma_{xy} = 0$$

This follows from translational symmetry (clean limit). Uniform field couples only to COM momentum and cannot depend on e-e interactions.

In the clean limit, e-e interactions can enter through

- (i) Finite size effects (small spontaneous dc Hall effect due to edges)
- (ii) probing system at finite wavelength q
- In 2D limit, Goryo and Ishikawa (1999), found $\sigma_{xy} = c_{xy} \frac{c_s^2 k^2}{\omega^2 c^2 k^2}$
- 3D case: Lutchyn, Nagornykh, Yakovenko and Roy, Kallin, PRB (2008).

With disorder, translational invariance is broken and relative and COM degrees of freedom couple → nonzero Kerr effect is allowed.



External vertices give $\sigma_{ij}p_iq_j$ $\rightarrow pxq = pqsin\theta$ Angular integral averages to zero

Goryo identified diagrams of order $n_i U^3$ (skew scattering) which contribute.

 \rightarrow sin² $\theta \rightarrow$ nonzero, only for I=1 !

 \sim comparable to experiment; a bit

 $Tr[\tau_3G(p)G(k)]=2i\Delta^2sin\theta$

small & likely to be reduced

$$\sigma_{xy}^{(v)}(\omega) = \gamma_{BCS}^2 \left(1 - \frac{T}{T_c}\right) \frac{l_1}{\xi_0} \left(\frac{\epsilon_F}{\pi \tau_0}\right)^{3/2} \frac{\sigma_{xy}^{(0)}}{(\omega + i/\tau_0)^3}$$

Explanation? Test by studying disorder and frequency dependence of Kerr effect.

Jun Goryo, PRB (2008).

Phase sensitive measurements

AuIn-Sr₂RuO₄ SQUID - Nelson et al, Science (2004)



Tunneling from singlet to triplet allowed by spin-orbit coupling Geshkenbein, Larkin, Barone (GLB) geometry

 $j_{\rm s} \propto \langle \operatorname{Re}(c_{21}s_{21}^*) \operatorname{Im}[\Psi^* \mathbf{d} \cdot (\mathbf{n} \times \mathbf{k})] \rangle_{\rm FS}$

Phase ~ d·(n x k) gives π for opposite b-c faces (GLB) and zero for same b-c faces (SS)



Dashed lines show zero field corrected for induced flux which varies with T

Recent phase-sensitive measurements



SRO single crystal (black) with 4 Josephson junctions. Gray ribbons are Pb thin film counterelectrodes. A field B is applied along c-axis.

Ideal junction of area A gives Fraunhofer pattern:

 $I_{\rm c}(\Phi) = J_{\rm c}A \frac{\sin(\pi \Phi/\Phi_0)}{\pi \Phi/\Phi_0}$

Observe "Fraunhofer-type patterns" which they compare to currents expected from relative phases of 0, π , $\pm \pi/2$. They conclude one needs all four relative phase factors to best model the data. Also see dynamics, which they model as dynamic domains with average size of ~1 micron.

F. Kidwingira, J.D. Strand, D.J. Van Harlingen, and Y. Maeno, Science 314, 1271 (2006).





Observed "complicated modulations characteristic of interference between regions with different phase and size..."

Data is best captured in the modeling of dynamic ±p±ip domains.

Domain walls which intersect surface at an angle (not $\pi/2$) can give rise to phase shifts other than ± 1 . (Sigrist)

F. Kidwingira, J.D. Strand, D.J. Van Harlingen, and Y. Maeno, Science 314, 1271 (2006).

Time Reversal Symmetry Breaking

Experiment	TRSB?	Domain size [limit >0.3µ]
muSR (Luke & Ishida)	Yes	< 2 µ
Kerr rotation (Kapitulnik)	Yes	> 50 μ with field cooling ~ > 15-20 μ in ZFC
Scanning Hall Probe (Moler)	No	< 1 µ
SQUID (Kirtley)	No	< 2 µ
Tunneling (van Harlingen)	Yes	<1µ ~0.5µ dynamic
Tunneling (Liu) Corner junctions	Parity Yes	>10-50 µ

Are there nodes?

Penetration Depth



Low T data shows power law behaviour \rightarrow nodes?

Analysis complicated by multiband effects which give powerlaw-like behaviour down to relatively low T. Need very low T data (below 0.05K) to resolve differences.

Data is inconclusive.

Comparison of data, fit with line nodes, fit with no nodes

From Kusunose & Sigrist Euro Phys Lett (2002); Data from Bonalde et al. PRL (2000)

Putting it all together

- SRO is an unconventional SC; likely triplet pairing (JJ expt also points to triplet) but NMR data with B along c-axis is not explained.
- Substantial evidence of time-reversal symmetry breaking, but all require assumptions about domain walls and/or disorder, and absence of observable edge currents remains a puzzle.
- Need direct evidence of domain walls (if they exist!)
- Other order parameters?

The end